

THREE DIMENSIONAL GEODETIC INVERSION METHOD FOR  
STRESS MODELLING IN THE LITHOSPHERE

BY

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Submitted to the Department of Earth and Planetary Sciences  
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ABSTRACT

The goal of the present project is to determine the distribution of incremental stress inside the earth under a seismic region as a basis for quantitative studies of stress-induced earthquake precursors. We formulated an inverse problem for incremental stress in the earth to be determined from three-component displacements observed on the free surface. A body was cut out of the earth under the surface at which displacements are known from geodetic measurements.

A special finite-element method is designed to give a unique determination of stress in the interior of the volume from known surface displacements and the free surface condition. The scheme was successfully tested using artificial data for a point source buried in a homogeneous half-space. We are currently applying it to the actual data from southern California geodimeter network and levelling data during the Palmdale uplift episode. A preliminary result indicates that the state of stress at the depth of a few km can be considerably different from the horizontal stress measured on the surface. The estimated incremental stress shows an encouraging agreement with observations on geomagnetic field and earthquake swarm activities.

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## Chapter 1. INTRODUCTION

A precise knowledge of the state of stress in the lithosphere in a seismically active region is desirable since the stress is believed to be the cause of an earthquake.

Many studies have been done to infer the state of stress in the lithosphere. Laboratory studies suggest high shear stress up to 2 kb along the San Andreas fault (Stesky and Brace, 1973, Stesky, 1975), while heat flow studies give low upper limit around 250 bar for possible shear stress there (Brune et al., 1969, Lachenbruch and Sass, 1973). This low shear stress is consistent with the value suggested from the studies of driving force for plate tectonics (Forsyth and Uyeda, 1975, Richardson and Solomon, 1977, Richardson, 1978), and seismic studies on stress drop (Aki, 1966, Wyss, 1970, Wyss and Molnar, 1972), although recent discussion on plate tectonics and earthquake stress-drop (Hanks, 1977) suggest high shear stress of the order of a kilobar.

The lack of agreements among these studies show a fundamental difficulty to know the state of stress in the lithosphere precisely. On the other hand the stress increment in the lithosphere may be easier to estimate than absolute stress since repeated geodetic measurements can give the incremental displacements at the surface.

The incremental stress may be related to earthquake precursors in many ways. Sassa and Nishimura (1956) reported rapid tilt changes in which so-called S-shaped changes in

the tilt-vector diagram were observed to occur a few hours prior to the Nanki earthquake of 1950. The magnitude of tilt was of the order of 0.1" at a distance of 100 km from the epicenter. They observed similar tilt changes prior to some other earthquakes. There have been many precursory anomalous changes in land level such as reported for the Niigata earthquake of 1964 (Danbara, 1973). Castle et al. (1974) studied levelling data near San Fernando and found that anomalous level changes with maximum value of 200 mm had taken place in a few years preceding the San Fernando earthquake of 1971. An aseismic creep along the fault at depth or a dilatancy are considered as causing these land deformations prior to earthquakes though exact mechanism is not known (Wyss, 1977; Thatcher, 1976). These land deformations are believed to be one of the most promising precursors for earthquakes because they have been frequently observed prior to many shallow earthquakes (Rikitake, 1975, 1976).

On the other hand, other precursors such as changes in  $V_p/V_s$ , resistivity, geomagnetic field are still under debate although they are also quite promising precursors, especially in view of the dilatancy-diffusion hypothesis (Nur, 1969). After Semenov (1969) observed that the ratios of travel times of S and P waves significantly varied prior to earthquakes in Russia, this has been followed by both verifications and contradictions. For example Whitcomb

et al. (1973) reported 10% change in  $V_p/V_s$  over the three years prior to the San Fernando earthquake of 1971 and Stewart (1973) showed a  $V_p$  decrease by 20% prior to the Pt. Mugu earthquake of 1973 while no change in  $V_p/V_s$  was observed in the Bear Valley earthquake of 1972 (Bakun et al., 1973). Bakun et al. attributed their negative results to the stress level at shallow depths which might be too low for dilatancy to take place. The laboratory experiments showed that volumetric strain necessary to explain the observed large  $V_p/V_s$  change was much greater than the stress level expected in the Bear Valley (Hadley, 1975).

Electrical resistivity change up to 24% was observed two months before the Bear Valley earthquake of 1972 by Mazzella and Morrison (1974). Laboratory experiments showed dramatic changes in the electrical properties of rock prior to failure (Brace and Orange, 1978a) in agreement with some field observations. On the other hand, a theoretical study of resistivity change based on a model of strike slip fault showed that the observed resistivity changes were several orders of magnitude larger than predicted for the expected stress change.

The geomagnetic change due to the piezomagnetic effect of rock under incremental stress is known to be very effective as a precursor as well as an indirect way to estimate the incremental stress at depth. Theoretical



study shows, assuming optimum material constants such as high magnetization and high stress sensitivity, the stress change caused by slip on a fault at shallow depth is sufficient to produce the geomagnetic field change observed on the surface (Johnston, 1978, Johnston et al., 1979).

The discrepancy between the theory on the stress-induced precursory phenomena based on laboratory data and precursory phenomena observed in the field may be attributed to the scale effect of specimen as well as the state of incremental stress at depth. The purpose of this thesis work is to develop an inversion method based on the finite element method using geodetic data on three-component displacement obtained at the earth's surface for finding the distribution of incremental stress at depths in a seismically active region. Such a distribution of incremental stress in the lithosphere will be useful together with laboratory observation on rock properties (Brace and Orange, 1968a, 1968b, Nur, 1969, Brace, 1972, Hadley, 1975, Stesky, 1975, Johnston, 1978) to find the cause of precursory phenomena. We applied the inversion method to the Palmdale uplift, southern California and obtained some preliminary encouraging results.

## Chapter 2. FINITE ELEMENT METHOD

### 1. General Formulation

In order to develop a finite element scheme for elasticity problems, we have to decide the type of generalized coordinate and the order of interpolation function as well as the shape of an element. If we use nodal displacements as a generalized coordinate, we have so-called finite element displacement method based on minimum potential energy principle. If we choose stress as a generalized coordinate, we have so-called conjugate finite element method based on minimum complementary energy principle. If we use displacements together with stress as a generalized coordinate, we have so-called hybrid finite element method. The last one seems most adequate for applying to our inversion, but the theory as well as numerical technique of this method is still not well established, and therefore we need further study for applying it to our inverse problem (Tong and Rosett, 1978, Oden and Reddy, 1976). Since fairly well established theory and numerical technique exist for the displacement finite element method, we shall formulate our finite element scheme and inversion scheme according to this method.

We shall define displacements within an element in a matrix form as

$$[U] = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (1)$$

where  $u, v, w$  are displacements in  $x, y, z$  direction. This displacement vector is presented in terms of interpolation functions and the generalized coordinate, i.e., nodal displacements in the displacement based finite element method. In matrix, they are:

$$[U] = [D(x, y, z)] [U^n] \quad (2)$$

where  $[U]$  is displacement vector, defined before,  $D(x, y, z)$  is interpolation matrix which gives  $[U]$  at an arbitrary point from the nodal displacement matrix  $[U^n]$ .  $[U^n]$  can be written explicitly as

$$[U^n]^T = [u_1 v_1 w_1 \ u_2 v_2 w_2 \dots u_k v_k w_k] \quad (3)$$

where  $u_i, v_i, w_i$  are  $x, y, z$  components of displacement vector at  $i$ th node. The value of  $k$  depends on the degree of interpolation function and shape of an element, and  $k=20$  if parabolic interpolation is used in a hexahedral element.  $T$  denotes transpose of a matrix.

We shall define strain tensor  $[e]$ , traction vector  $[T]$ , force vector  $[F]$ , and elasticity matrix  $[C]$ . They are:

$$[e]^T = [\epsilon_{xx} \ \epsilon_{yy} \ \epsilon_{zz} \ \gamma_{xy} \ \gamma_{xz} \ \gamma_{yz}] \quad (4)$$

$$[T]^T = [T_x \ T_y \ T_z] \quad (5)$$

$$[F]^T = [F_x \ F_y \ F_z] \quad (6)$$

where  $\epsilon_{ij}$ ,  $\gamma_{ij}$  are six components of a strain tensor in which we use engineering shear strain.  $T_i$  or  $F_i$  ( $i = x, y, z$ ) are the  $x, y, z$  components of surface traction vector and force vector. The elasticity matrix is in case of isotropic material,

$$[C] = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \quad (7)$$

where  $\lambda$ ,  $\mu$  are Lamé's constants.

We can define strain matrix  $[e]$  in terms of strain-transformation matrix  $[E]$  and nodal displacement matrix. This is:

$$[e] = [E] [U^n] \quad (8)$$

$$[E] = \begin{bmatrix} \frac{\partial}{\partial x}[D] & 0 & 0 \\ 0 & \frac{\partial}{\partial y}[D] & 0 \\ 0 & 0 & \frac{\partial}{\partial z}[D] \\ \frac{\partial}{\partial y}[D] & \frac{\partial}{\partial x}[D] & 0 \\ \frac{\partial}{\partial z}[D] & 0 & \frac{\partial}{\partial x}[D] \\ 0 & \frac{\partial}{\partial z}[D] & \frac{\partial}{\partial y}[D] \end{bmatrix} \quad (9)$$

where  $[D]$  was defined in [2]. The total potential energy of an elastic body can be presented as (Tong and Rosett, 1978)

$$\pi = \int_V \frac{1}{2} [e]^T [C] [e] dv - \int_V [U]^T [F] dv - \int_V [U]^T [T] ds \quad (10)$$

where  $\pi$  is total potential energy,  $s$  is boundary surface and integration covers whole volume of a body. This is the quantity to be minimized.

The above potential energy can be expressed as a sum of energy for each element, i.e.,

$$\pi = \sum_{n=1}^m \pi_n \quad (11)$$

where  $m$  is the total number of elements. Substitution of eq. (8) into eq. (10) yields

$$\pi_n = \frac{1}{2} [U^n]_n^T [K]_n [U^n]_n - [U^n]_n^T [R]_n \quad (12)$$

where  $[ ]_n$  represents  $n$ th element, and  $[K]_n$  and  $[R]_n$  are element stiffness matrix and element load matrix. Explicitly they are

$$[K]_n = \int_V [E]_n^T [C] [E]_n dv \quad (13)$$

and

$$[R]_n = \int_V [D]^T [F] dv + \int_S [D]^T [T] ds. \quad (14)$$

Making the potential energy in each element, presented by eq. (12) stationary with respect to generalized coordinate, i.e., nodal displacement, we have a linear equation for each element as:

$$[K]_n [U^n]_n = [R]_n . \quad (15)$$

## 2. Isoparametric element

The choice of an interpolation function is an important part of the finite element method and depends on the type of element to be used and on desired accuracy. The concept of isoparametric element developed by Zienkiewicz, and his associate [Zienkiewicz,1972] is one of the most versatile interpolations because of its flexibility.

The basic concept in the isoparametric element formulation is to express the element coordinates and its displacements by the same interpolation function using the natural coordinate system of the element. The coordinate interpolations in a three-dimensional element are as:

$$\begin{aligned}
 x &= \sum_{i=1}^{\ell} h_i x_i \\
 y &= \sum_{i=1}^{\ell} h_i y_i \\
 z &= \sum_{i=1}^{\ell} h_i z_i
 \end{aligned}
 \tag{16}$$

where  $x, y, z$  are the coordinates at any point of the element and  $x_i, y_i, z_i$  ( $i = 1, \ell$ ) are the coordinates of the  $i$ th node in the element, and  $\ell$  denotes the total number of nodes in the element.

The unknown quantities in eq. (16) i.e. the interpolation function  $h_i$ , have the fundamental property that its value in the natural coordinate system is unity at the  $i$ th node and is zero at all other nodes. Using this condition the interpolation function  $h_i$  corresponding to a specific nodal point configuration can be derived. In the case of our three-dimensional 20 nodes parabolic interpolation in hexahedral element, these interpolation functions are given by the following equations.

$$h_1 = \frac{1}{8} (1+r) (1+s) (1+t) (r+s+t-z)$$

$$h_2 = \frac{1}{8} (1-r) (1+s) (1+t) (-r+s+t-z)$$

$$h_3 = \frac{1}{8} (1-r) (1-s) (1+t) (-r-s+t+z)$$

$$h_4 = \frac{1}{8} (1+r) (1-s) (1+t) (r-s+t-z)$$

$$h_5 = \frac{1}{8} (1+r) (1+s) (1-t) (r+s-t-z)$$

$$h_6 = \frac{1}{8} (1-r) (1+s) (1-t) (-r+s-t-z)$$

$$h_7 = \frac{1}{8} (1-r) (1-s) (1-t) (-r-s-t-z) \quad (17)$$

$$h_8 = \frac{1}{8} (1+r) (1-s) (1-t) (r-s-t-z)$$

$$h_9 = \frac{1}{4} (1-r^2) (1+s) (1+t)$$

$$h_{10} = \frac{1}{4} (1-r) (1-s^2) (1+t)$$

$$h_{11} = \frac{1}{4} (1-r^2) (1-s) (1+t)$$

$$h_{12} = \frac{1}{4} (1+r) (1-s^2) (1+t)$$

$$h_{13} = \frac{1}{4} (1-r^2) (1+s) (1-t)$$



$$h_{14} = \frac{1}{4} (1-r) (1-s^2) (1-t)$$

$$h_{15} = \frac{1}{4} (1-r^2) (1-s) (1-t)$$

$$h_{16} = \frac{1}{4} (1+r) (1-s^2) (1-t)$$

(17) (cont'd)

$$h_{17} = \frac{1}{4} (1+r) (1+s) (1-t^2)$$

$$h_{18} = \frac{1}{4} (1-r) (1+s) (1-t^2)$$

$$h_{19} = \frac{1}{4} (1-r) (1-s) (1-t^2)$$

$$h_{20} = \frac{1}{4} (1+r) (1-s) (1-t^2)$$

where  $r, s, t$  are natural coordinate whose values range from -1 to 1 as shown in Fig. 1.

The displacements in an element are interpolated by the same relation as the coordinates in the isoparametric interpolation, i.e., for displacement at any point of the element, we have:

$$u = \sum_{i=1}^{\ell} h_i u_i$$

$$v = \sum_{i=1}^{\ell} h_i v_i \quad (18)$$

$$w_i = \sum_{i=1}^{\ell} h_i w_i$$

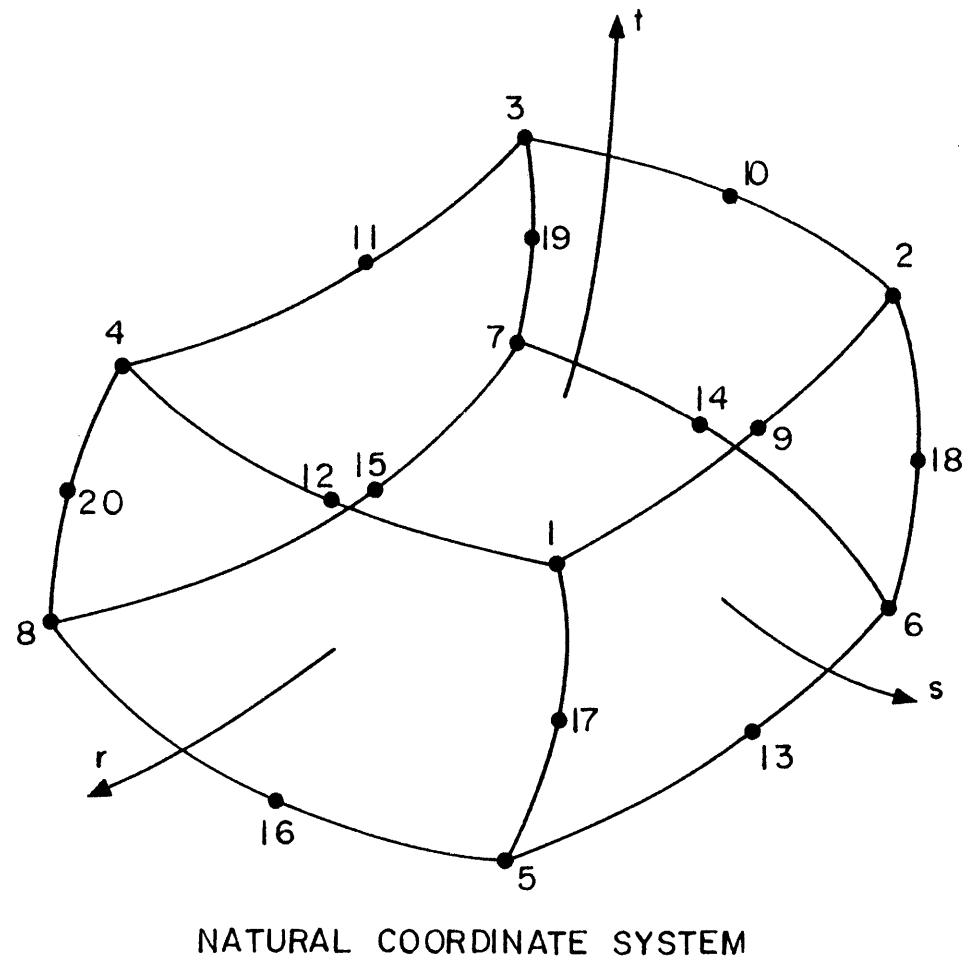
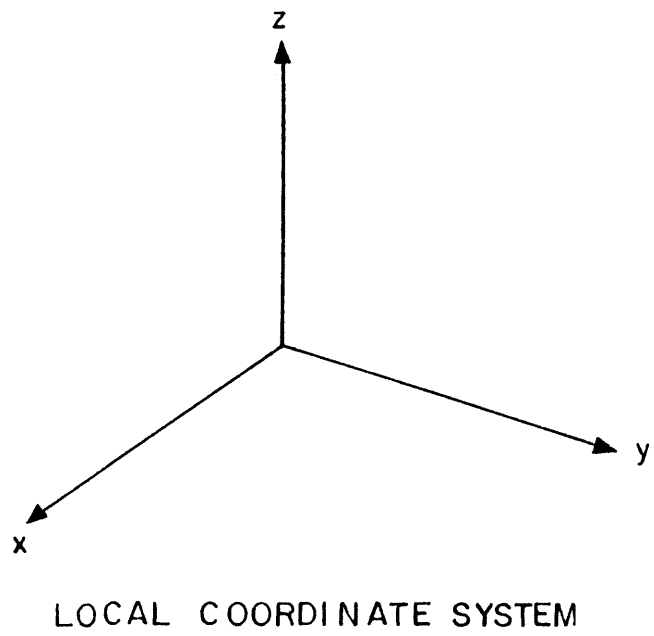


Fig. 1 Isoparametric 20 nodes element

where  $u, v, w$  are displacement component at any point in the element and  $u_i, v_i, w_i$  ( $i = 1, \ell$ ) are these components of nodal displacements of the element.

In order to evaluate the stiffness matrix  $[K]$  of an element, we need to calculate the strain-transformation matrix  $[E]$ . Since the element displacements are presented in the natural coordinate system as shown in eq. (17) we must relate the derivatives in the local  $x, y, z$  coordinates to the derivatives in the  $r, s, t$  natural coordinates. Using a chain rule for partial derivatives, the relations are:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial}{\partial t} \frac{\partial t}{\partial x}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial}{\partial t} \frac{\partial t}{\partial y} \quad (19)$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial r} \frac{\partial r}{\partial z} + \frac{\partial}{\partial s} \frac{\partial s}{\partial z} + \frac{\partial}{\partial t} \frac{\partial t}{\partial z} .$$

These equations show that we need the explicit inverse relationship between  $x, y, z$  and  $s, r, t$ , i.e. we need to know the functions

$$r = f_1(x, y, z)$$

$$s = f_2(x, y, z) \quad (20)$$

$$t = f_3(x, y, z)$$

which are generally difficult to obtain explicitly and it is natural recourse to use a numerical procedure.

The relation between the derivatives in the  $r, s, t$  coordinate system and derivatives in the  $x, y, z$  coordinates system can also be written using the chain rule:

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial}{\partial z} \frac{\partial z}{\partial r}$$

$$\frac{\partial}{\partial s} = \frac{\partial}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial}{\partial z} \frac{\partial z}{\partial s} \quad (21)$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial}{\partial z} \frac{\partial z}{\partial t}$$

in matrix form we have

$$\frac{\partial}{\partial[r]} = [J] \frac{\partial}{\partial[x]} \quad (22)$$

where  $[J]$  is the Jacobian operator matrix i.e.

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t} \end{bmatrix} \quad (23)$$

Since we can evaluate Jacobian operator matrix  $[J]$  explicitly using eq. (17), we can get the inverse relationship between derivatives in two coordinate systems as

$$\frac{\partial}{\partial[x]} = [J]^{-1} \frac{\partial}{\partial[r]} \quad (24)$$

provided that matrix  $[J]$  is non-singular. This condition is satisfied if the mapping between these two coordinate systems, i.e. eq. (20) is one to one.

Using eq. (18) and eq. (24) we evaluate derivatives needed to construct the strain transformation matrix  $[E]$ .

The element stiffness matrix corresponding to the local degree of freedom is derived as

$$[K] = \int_V [E]^T [C] [E] dv \quad (25)$$

where  $[C]$  is elasticity matrix defined before.

Since strain-transformation matrix  $[E]$  is a function of the natural coordinate  $r, s, t$ , the volume integral  $dv$  should be written in terms of this coordinate system. That is:

$$dv = \det [J] dr ds dt \quad (26)$$

where  $\det$  denotes determinant of Jacobian operator matrix. Since explicit solution for  $[J]^{-1}$  seldom exists, we must use numerical integration to evaluate the volume integral in eq. (25). We have

$$[K] = \sum_{i=1}^m \sum_{j=1}^m \sum_{k=1}^m \alpha_i \alpha_j \alpha_k [E]_{ijk}^T [C] [E]_{ijk} \det[J]_{ijk} \quad (27)$$

where  $[ ]_{ijk}$  represents a quantity evaluated at a point  $r_i, s_j, t_k$ ,  $m$  is the total number of sampling points for numerical integration, and  $\alpha_i, \alpha_j, \alpha_k$  are weight at these points. The sampling points and weighting factors depend on the integration scheme to be used. We used Gauss-Legendre quadrature method, for which the sampling points and weights are shown in Fig. 2 and Table 1.

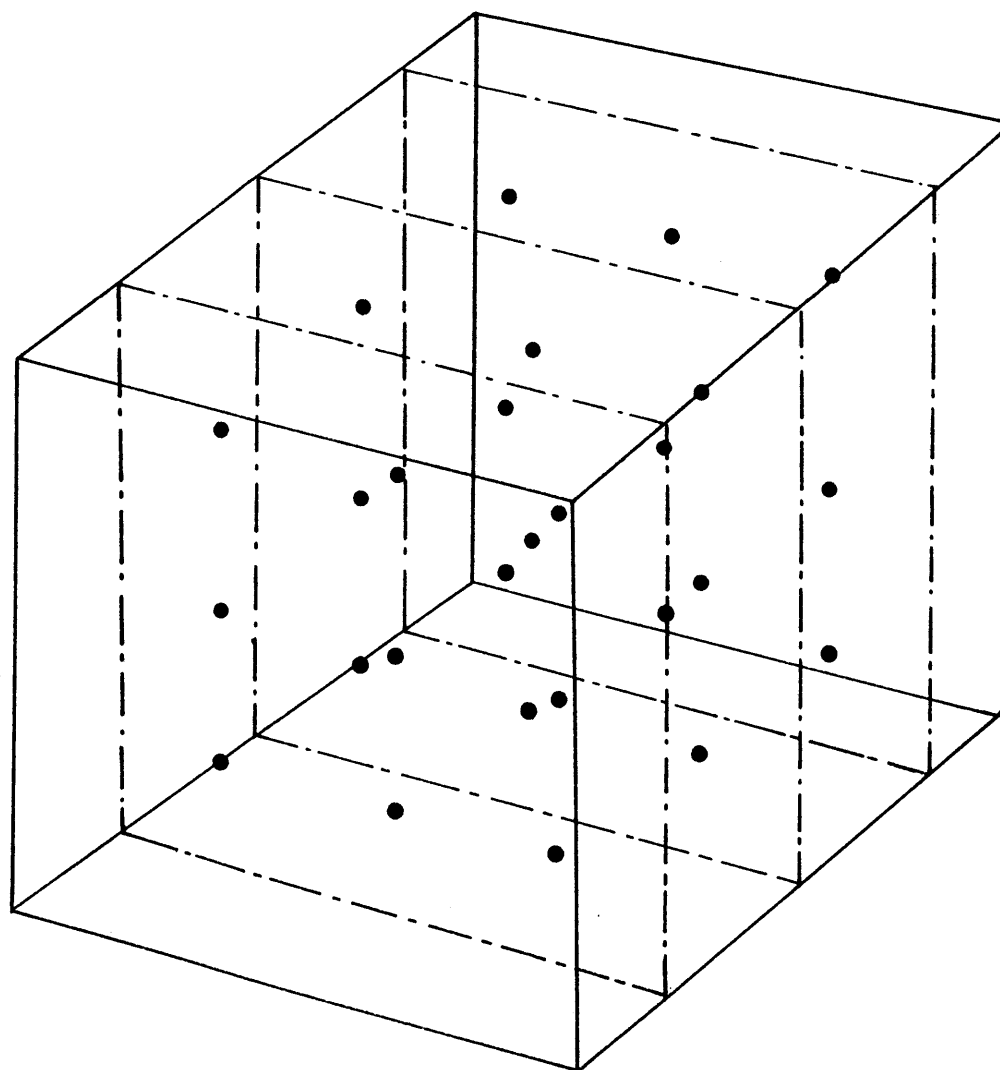


Fig. 2 Sampling points for numerical integration

### 3. Assembling Matrix for the Whole Body

The final stage of finite element method is to assemble  $[K]_n$  matrix and  $[R]_n$  matrix in each element to global matrix  $[K]$  and  $[R]$  to attain the equilibrium condition of the whole body. Since matrix  $[K]$  is a transfer matrix which represents contributions of each component of nodal displacement matrix  $[U_n]$  to components of the load matrix  $[R]$ , this is nothing but a summation and re-numbering of matrix for each element.

For example, if we re-number the  $i$ th node of the  $\ell$ th element as the  $m$ th node of the global system, and the  $j$ th node of  $\ell$ th element as the  $n$ th node of the global system, we have

$$[K_{mn}]_g = \sum_{\ell} [K_{ij}]_{\ell},$$

$$[R_m]_g = \sum_{\ell} [R_i]_{\ell}, \quad (28)$$

$$[R_n]_g = \sum_{\ell} [R_j]_{\ell},$$

where  $[K_{ij}]$  is  $ij$ -component of stiffness matrix  $[K]$  and subscript  $g$  represents global matrix and the summation is taken over elements  $\ell$  sharing the particular node. After assembling whole element matrix using eq. (28) we have linear equations to be solved, i.e. we have

$$[K] [U] = [R]. \quad (29)$$



### Chapter 3. GEODETIC INVERSION SCHEME

#### 1. General Theory

The use of the finite element method in inverse problem, especially in parameter estimation problem, has been studied in many fields in science and engineering during the last decade. In geophysics some inversions based on the finite element method were studied. Most of them used the observed surface displacements associated with an earthquake to estimate fault parameters such as slip vector (Jangle and Frazar, 1973), visco-elastic constant of surrounding material (Smith, 1973), and stress drop (McCowan et al., 1977) with specified boundary conditions.

In our geodetic inverse problem, we want to calculate the stress and displacements inside the earth's crust without specifying anything on the internal boundary inside the earth. The only boundary conditions available to us are displacements and free surface condition on the surface boundary. Mathematically this is known as a Cauchy boundary condition for an elliptic equation and in general results in unstable solutions for the entire region (Morse and Feshbach, 1953).

Suppose that the problem is reduced to a linear operator equation  $Lx = y$ , where  $x$  is the desired solution. When this problem is ill-posed, there are basically two methods for remedy (Lunz, 1979). One is known as the regularization

method in which we use a priori choice of the space of the permitted solution to avoid instability by modifying the functional to be minimized. For example, we minimize

$$J(x) = ||Lx - y|| + \alpha \psi(x) \quad (30)$$

instead of solving the equation  $Lx = y$  directly where  $|| \quad ||$  denotes an appropriate norm,  $\alpha > 0$  and  $\psi$  is a functional defined on a solution space. This method is equivalent to the damped least squares (Levenbergs, 1944), the stochastic inverse (Franklin, 1970) and the generalized inverse (Wiggins, 1972) depending on the property of  $\psi$  and the norm used.

Another way to solve this ill-posed operator equation is the so-called expansion method. In this method we choose finite dimensional space  $X_n$  and look for a solution in this space. If  $[\psi_{in}]$  denotes a basis for  $X_n$ , we look for a solution of the form

$$x_n = \sum_{i=1}^h \alpha_i \psi_{in} \quad (31)$$

such that

$$Lx_n \approx y. \quad (32)$$

The latter method seems more convenient than the former if we use the finite element method to discretize the operator  $L$ , because of a large amount of computation for calculating the norm. On the other hand, in the expansion method we can choose any solution space so that accuracy and economic factor can be satisfied at the same time. This can be attained by appropriate choice of expansion by eq. (31) with specific mesh configurations which assure the same number of knowns and unknowns.

## 2. Inversion Scheme

Using the finite element method derived in a previous chapter, we can have discretized form of the operator equation  $Lx = y$ . Written explicitly, this equation is in a partitioned form,

$$\begin{array}{c}
 \\
 \\
 k \\
 \ell \\
 m \\
 n
 \end{array}
 \begin{array}{c}
 k \quad \ell \quad m \quad n \\
 \left[ \begin{array}{cccc}
 K_{11} & K_{12} & K_{13} & K_{14} \\
 K_{21} & K_{22} & K_{23} & K_{24} \\
 K_{31} & K_{32} & K_{33} & K_{34} \\
 K_{41} & K_{42} & K_{43} & K_{44}
 \end{array} \right]
 \begin{array}{c}
 \left[ \begin{array}{c}
 U_1^* \\
 U_2^* \\
 U_3 \\
 U_4
 \end{array} \right]
 =
 \left[ \begin{array}{c}
 R_1^* \\
 R_2 \\
 R_3^* \\
 R_4
 \end{array} \right]
 \end{array}
 \end{array}
 \quad (33)$$

where  $K_{11}$  is a  $(k \times k)$  matrix.  $K_{12}$  is a  $(k \times \ell)$  matrix,  $K_{13}$  is a  $k \times m$  matrix,  $K_{14}$  is a  $(k \times n)$  matrix,  $K_{22}$  is a  $(\ell \times \ell)$  matrix,  $K_{23}$  is a  $(\ell \times m)$  matrix,  $K_{24}$  is a  $(\ell \times n)$  matrix,  $K_{33}$  is a  $(m \times m)$  matrix,  $K_{34}$  is a  $(m \times n)$  matrix,  $K_{44}$  is a  $(n \times n)$  matrix, and  $K_{21}$ ,  $K_{31}$ ,  $K_{32}$ ,  $K_{41}$ ,  $K_{42}$ ,  $K_{43}$  are transpose of  $K_{12}$ ,  $K_{13}$ ,  $K_{23}$ ,  $K_{14}$ ,  $K_{24}$ ,  $K_{34}$ , respectively.  $U_1^*$  is a  $(k \times 1)$  matrix,  $U_2^*$  is a  $(\ell \times 1)$  matrix,  $U_3$  is a  $(m \times 1)$  matrix.  $U_4$  is a  $(n \times 1)$  matrix,  $R_1^*$  is a  $(k \times 1)$  matrix,  $R_2$  is a  $(\ell \times 1)$  matrix,  $R_3^*$  is a  $(m \times 1)$  matrix, and  $R_4$  is a  $(n \times 1)$  matrix, and a star represents that elements of that matrix are known quantities. These known quantities are observed three components on the surface, stress free condition on the surface and the force balance condition inside the boundaries. The latter two conditions give zero values to the load matrix  $[R]$ .

The above matrix equation can be modified to give a more convenient form to analyze, that is:

$$\begin{bmatrix} K_{13} & K_{14} & 0 & 0 \\ K_{23} & K_{24} & -I & 0 \\ K_{33} & K_{34} & 0 & 0 \\ K_{43} & K_{44} & 0 & -I \end{bmatrix} \begin{bmatrix} U_3 \\ U_4 \\ R_2 \\ R_4 \end{bmatrix} = \begin{bmatrix} -K_{11} & -K_{12} & I & 0 \\ -K_{21} & -K_{22} & 0 & 0 \\ -K_{31} & -K_{32} & 0 & I \\ -K_{41} & -K_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1^* \\ U_2^* \\ R_1^* \\ R_3^* \end{bmatrix} \quad (34)$$

where  $[I]$  denotes a unit matrix of appropriate order. As can be seen from the above equation, our problem is overdetermined if  $\ell + m + 2n < \text{rank of } L_n = k + \ell + m + n$  underdetermined if  $k + \ell + m + n < \text{rank of } L_n = \ell + m + 2n$ , and ill-posed if  $\text{rank of } L_n < k + \ell + m + n$  and  $\text{rank of } L_n < \ell + m + 2n$ , and well-posed if  $\text{rank of } L_n = k + \ell + m + n = \ell + m + 2n$ , where  $L_n$  represents the matrix on L.H.S. of eq. (34).

We need, therefore, to have  $\text{rank of } L_n = k + \ell + m + n = \ell + m + 2n$  or  $k = n$ . This last condition shows that the number of surface nodes where we have both stress free condition and displacements observation (this excludes the boundary nodes on the surface) need to be the same as the number of boundary nodes inside the earth where we know neither tractions nor displacements. To attain this condition, we need to choose a special family of the finite element mesh configuration. This is mathematically equivalent to specify the base of solution space in expansion method as was shown in eq. (31).

The simplest mesh configuration which satisfies the condition is shown in Fig. 3. This mesh configuration has a certain advantage that it allows us to increase the order of interpolation function without changing the whole configuration. This is shown in Fig. 4. The values of  $k, \ell, m, n$  in cases of 8 nodes of interpolation and 20-nodes interpolation are presented in Table 2.

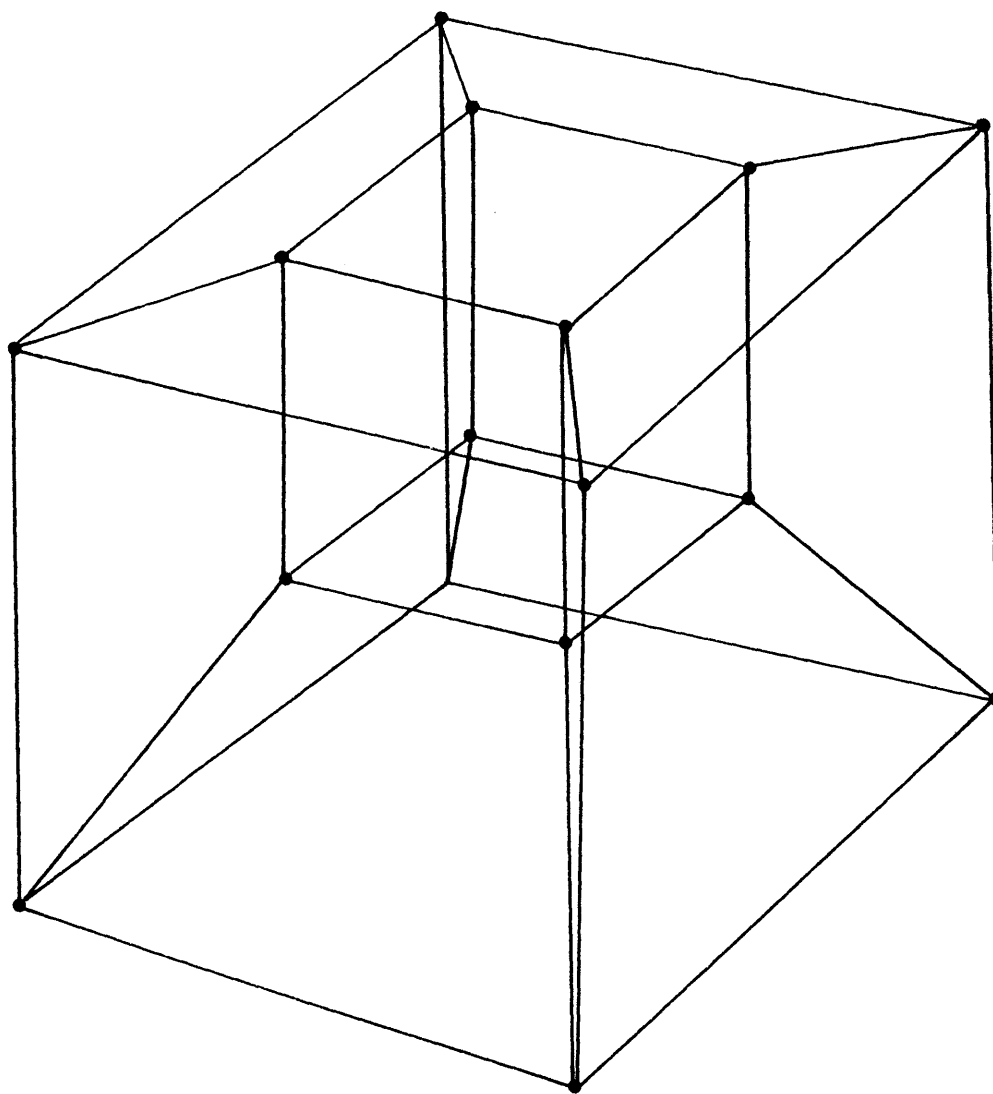


Fig. 3 Mesh configuration by 8 nodes element

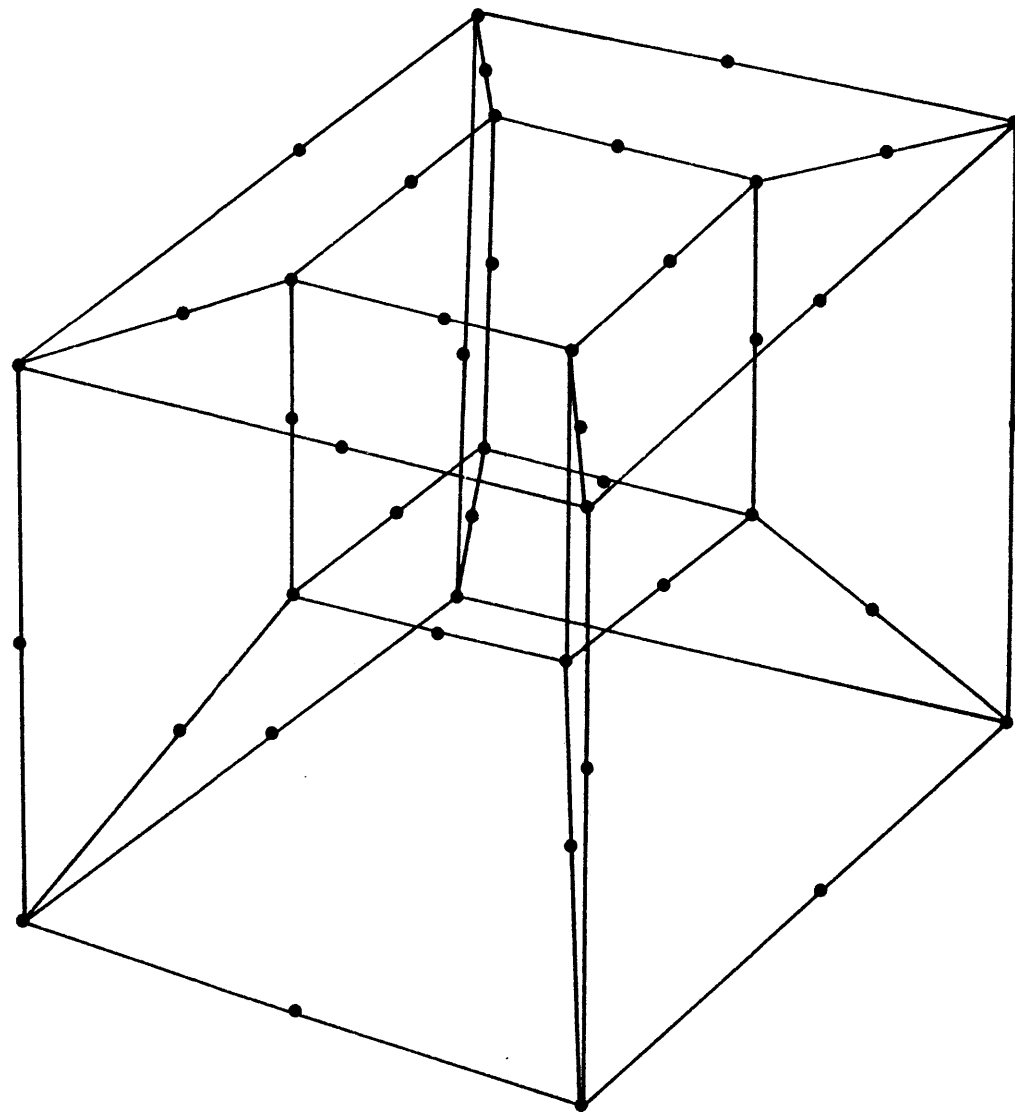


Fig. 4 Mesh configuration by 20 nodes element - I

Since in the parabolic interpolation using 20 nodes in an element, the highest term is square, we expect that a strain change in the element is at most linear. Thus if we have to approximate a rapidly changing stress field inside a block, the block must be further subdivided. We can improve the accuracy of solution by subdividing the inner block in Fig. 4 into 7 small blocks as shown in Fig. 5. The values of  $k$ ,  $\ell$ ,  $m$ ,  $n$  are also shown in Table 2. It is clear that the condition  $k = n$  still holds after this subdivision.

### 3. Error Analysis

There are two sources of error in this finite element inversion scheme. The first one is inherent to the property of matrix  $L_n$  in eq. (34). As shown earlier, for the problem to be well-posed it is necessary to have rank of  $L_n = k + \ell + m + n$  in addition to the condition  $k = n$ . This condition is satisfied, in principle, by the family of mesh configuration as shown previously, but numerically the system can be near rank-deficient.

To measure the well-posedness of the problem, it is convenient to use a condition number  $K$  defined as

$$K = \frac{\max || \lambda ||}{\min || \lambda ||} \quad (35)$$

where  $\lambda$  is an eigenvalue of matrix  $L_n$ .



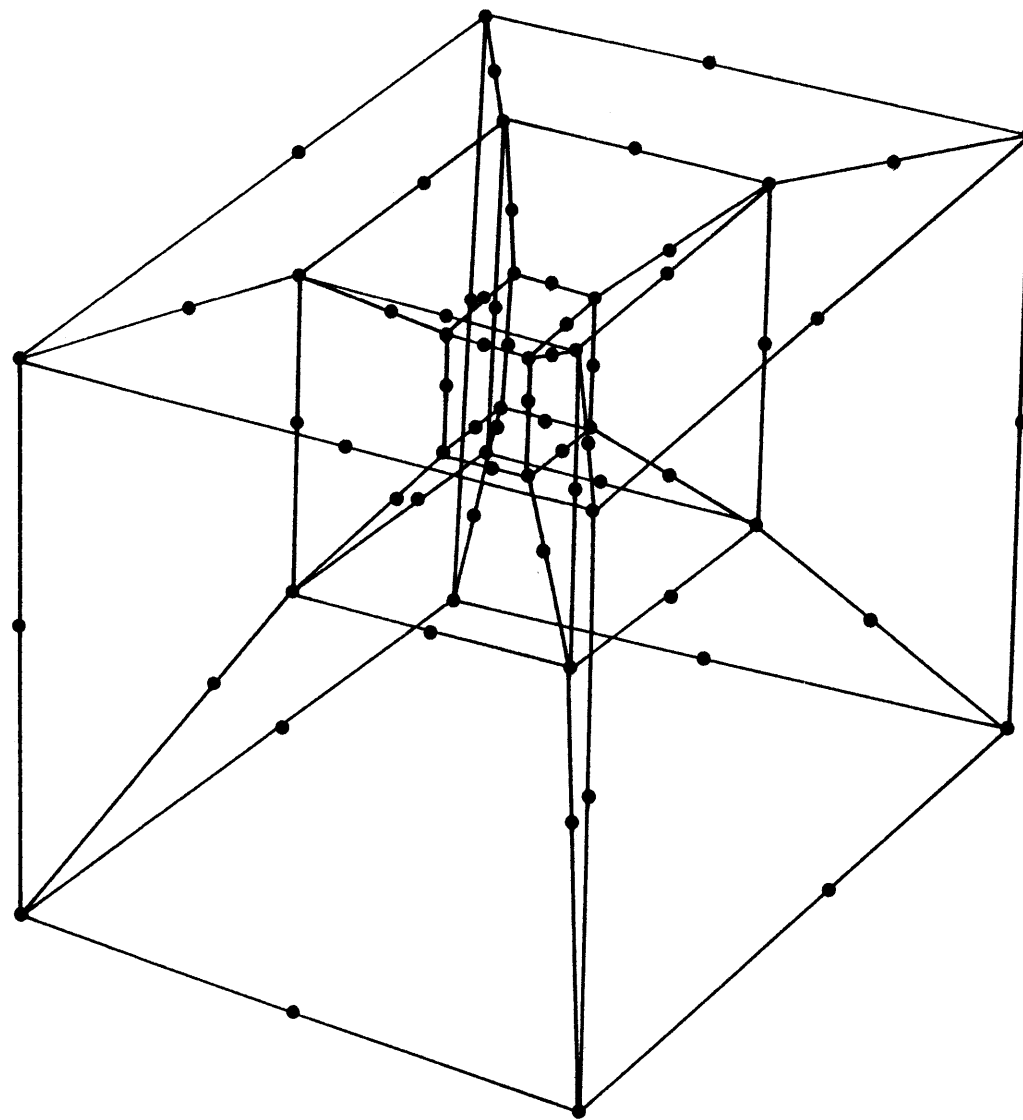


Fig. 5 Mesh configuration by 20 nodes element - II

For the block whose dimension is  $10 \times 10 \times 10$  km, condition number around  $10^5$  to  $10^6$  depending on the configuration of inner nodes is obtained by calculating the eigenvalue of matrix  $L_n$ . Using formula on the relation between the accuracy of fixed point arithmetic and condition number  $K$ , (Wilkinson, 1969) i.e.,

$$s \geq t - \log_{10} K \quad (36)$$

where  $s$  is the number of digits of precision in the solution and  $t$  is the number of digits of arithmetic used in the computer, we can have rough estimate on the truncation error involved in this finite element inversion. Eq. 36 shows that we have 1 digit to 0 digit of accuracies if we use 6-digit arithmetic in the computer. Therefore the use of double precision arithmetic is necessary.

The other source of error in our finite element inversion scheme is the error due to the use of specific mesh configuration with high order interpolation. Since the transformations of local coordinates  $x, y, z$  to natural coordinates  $r, s, t$  may be non-linear if the element is deformed from a rectangular as in Fig. 4, the error associated with the interpolation function may change considerably.

This situation becomes clear if we choose two-dimensional 8-nodes element as an example. (This element corresponds to a 20-nodes element in three-dimensions). This element is shown in Fig. 6. The transformation between  $(x, y)$  and  $(r, s)$

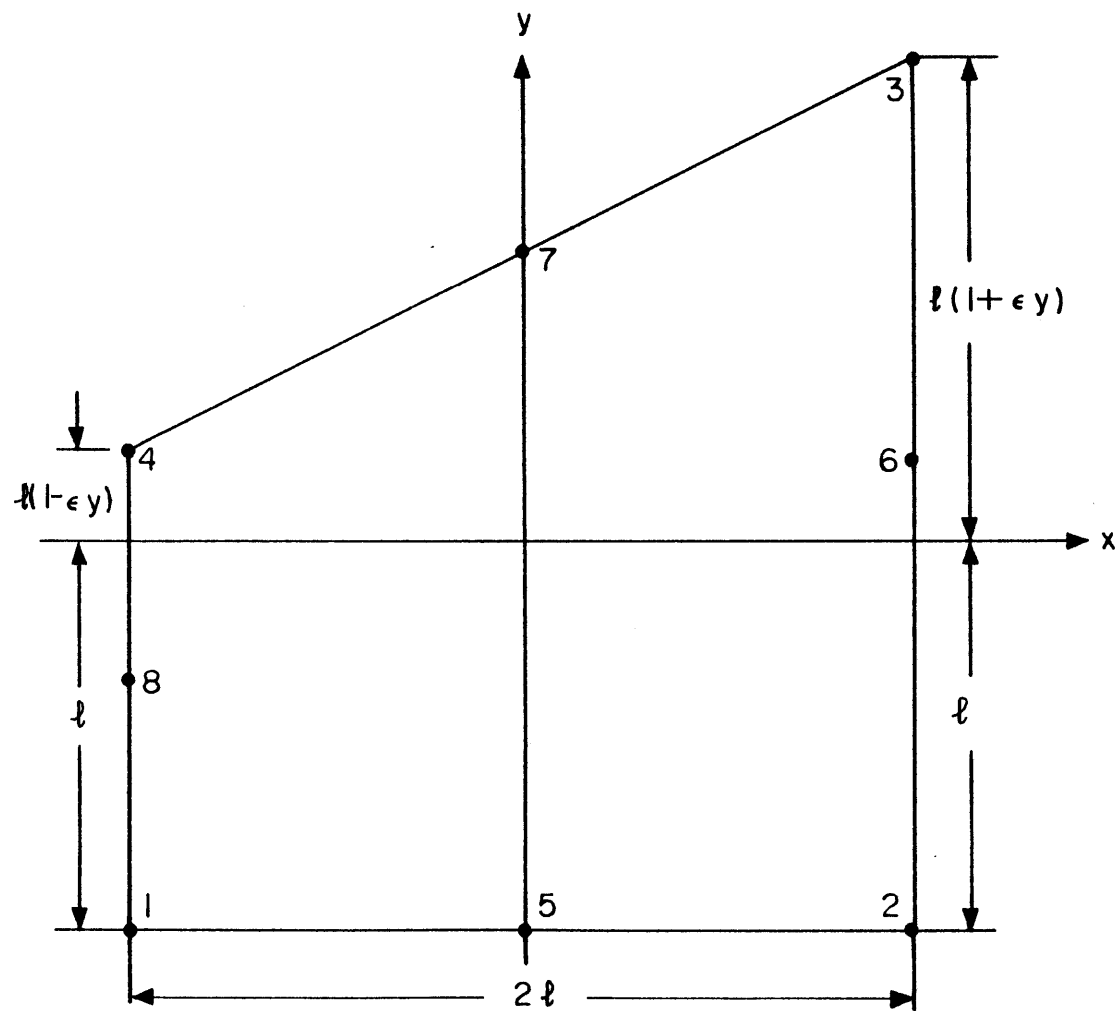


Fig. 6 Two-dimensional 8 nodes element

coordinates are simply,

$$\begin{aligned} x &= \ell r \\ y &= \ell \left\{ s + \frac{\epsilon_y}{2} r (1 + s) \right\} \end{aligned} \quad (37)$$

where  $\epsilon_y$  is the slope of upper edge shown in Fig. 6. For example, consider function  $\tilde{u} = y^2$ , which have nodal values  $\tilde{u}_i = y_i^2$ . The interpolation formula gives

$$\tilde{u} = \sum_{i=1}^8 h_i(r,s) y_i^2 \quad (38)$$

where  $h_i(r,s)$  is the interpolation function similar to eq. (19) to eq. (38).

Inserting eq. (11) into eq. (12), we obtain

$$\begin{aligned} \tilde{u} &= \sum_{i=1}^8 h_i(r,s) \ell^2 \left[ \left( s_i + \frac{\epsilon_y}{2} r_i (1 + s_i) \right)^2 \right] \\ &= \sum_{i=1}^8 h_i(r,s) \ell^2 \left( r_i + \frac{\epsilon_y}{2} s_i \right)^2 + \sum_{i=1}^8 2h_i(r,s) \ell^2 \left( s_i + \frac{\epsilon_y}{2} r_i \right) \\ &\quad r_i s_i \frac{\epsilon_y}{\ell^2} + \sum_{i=1}^8 h_i(r,s) + \frac{\ell^2 \epsilon_y^2}{4} r_i^2 s_i^2 \end{aligned} \quad (39)$$

Since the order of interpolation function  $h_i(r,s)$  is at most cubic in  $r$  and  $s$ , it is impossible to describe a function which contains a fourth power term in  $r$  and  $s$  exactly. Since the first two terms on RHS of equation (39) are parabolic and can be completely described, we have

$$\begin{aligned}
 u &= \ell^2 \left(s + \frac{\epsilon_Y}{2} r\right)^2 + \epsilon_Y^2 \left(s + \frac{\epsilon_Y}{2} r\right) rs + \sum_{i=1}^8 h_i(r,s) \frac{\ell^2}{4} \epsilon_Y^2 r_i^2 s_i^2 \\
 &= y^2 + \frac{\ell^2}{4} \epsilon_Y^2 \left( \sum_{i=1}^8 h_i(r,s) r_i^2 s_i^2 - r^2 s^2 \right) \quad (40)
 \end{aligned}$$

This equation shows that the error depends on  $\ell$  and  $\epsilon_Y$  and may be very sensitive to the mesh configuration.

The above error analysis gives us only a rough idea of the error involved. An optimum mesh configuration must be found by trial and error, since the complete error analysis for an optimization is too complicated. The mesh configuration shown in Fig. 7 appears to be near optimum for the case of a buried point force model used in our test.

#### 4. Testing With Artificial Data

In order to find the optimal mesh configuration, we used artificial data generated by a buried point force in a homogeneous half-space. Displacements at the free surface, known as Mindlin's solution (Mindlin, 1936), are used as an input to our inverse scheme. The calculated stress and displacements at depths by our inverse scheme are then compared with the true stress and displacement generated by the point force.

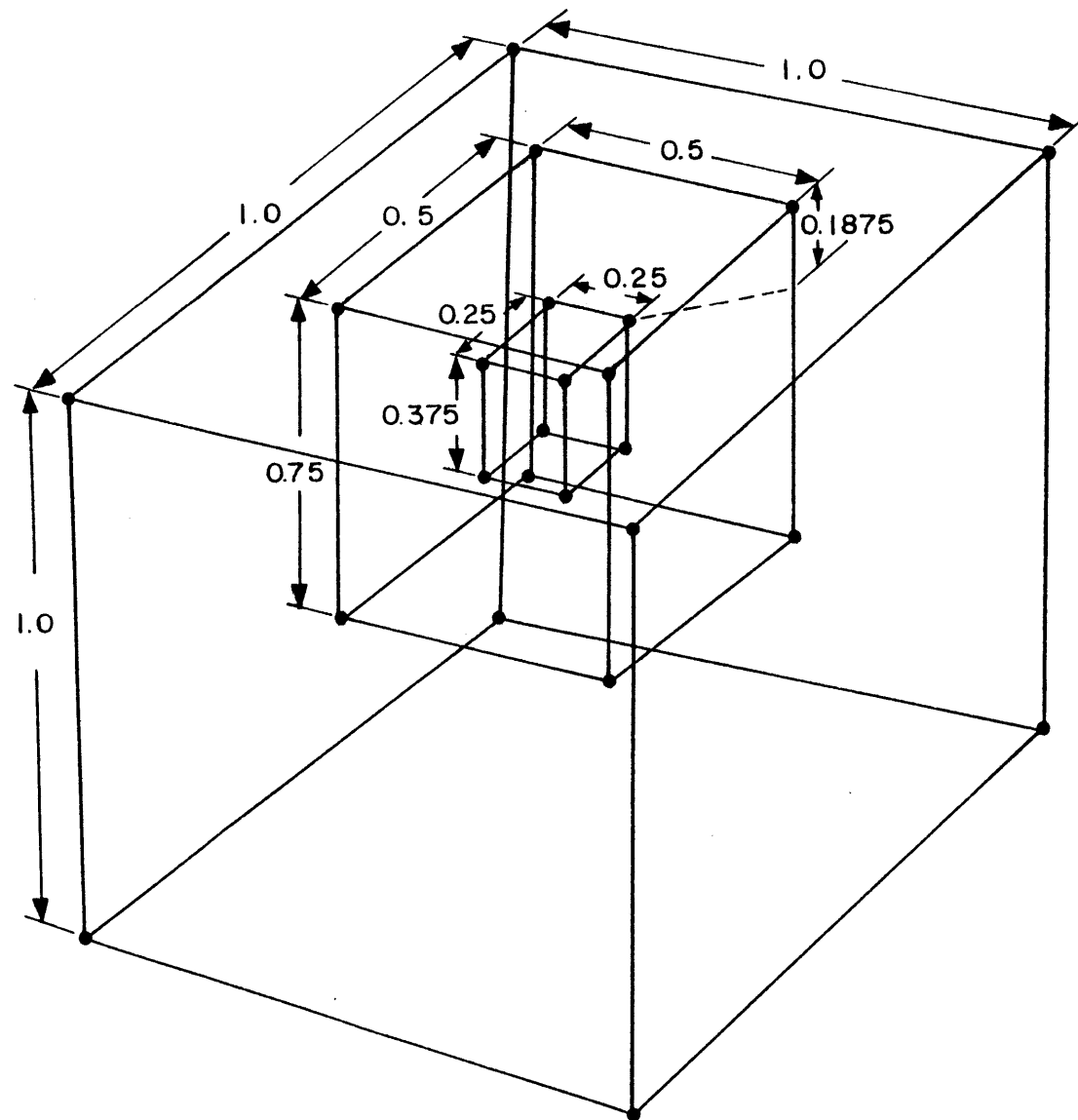


Fig. 7 Optimum mesh configuration

We used the configuration shown in Fig. 5, and the block size (outer boundary) was chosen to be  $10 \times 10 \times 10$  km or  $20 \times 20 \times 20$  km. The whole block was cut out of the half-space which contains a point force directed upward at depths 15 to 100 km. The horizontal location of block is also varied relative to the point force. An example of block location is depicted in Fig. 8. Figs. 9 and 10 represent the absolute error between the true displacement and stress in the center of each element and those obtained by our inversion scheme. Figs. 11 and 12 show the relative error between the true value and inverted one. These values are shown for various depths of the point source and a fixed horizontal distance (14 km). It is shown that for the point force at the depth of 25 km to 35 km, the accuracy of our inversion scheme is within 5% for displacement and within 30% for stress components. Compared with the sensitivity of error to the change of the depth of point source, the sensitivity to the horizontal distance is small.

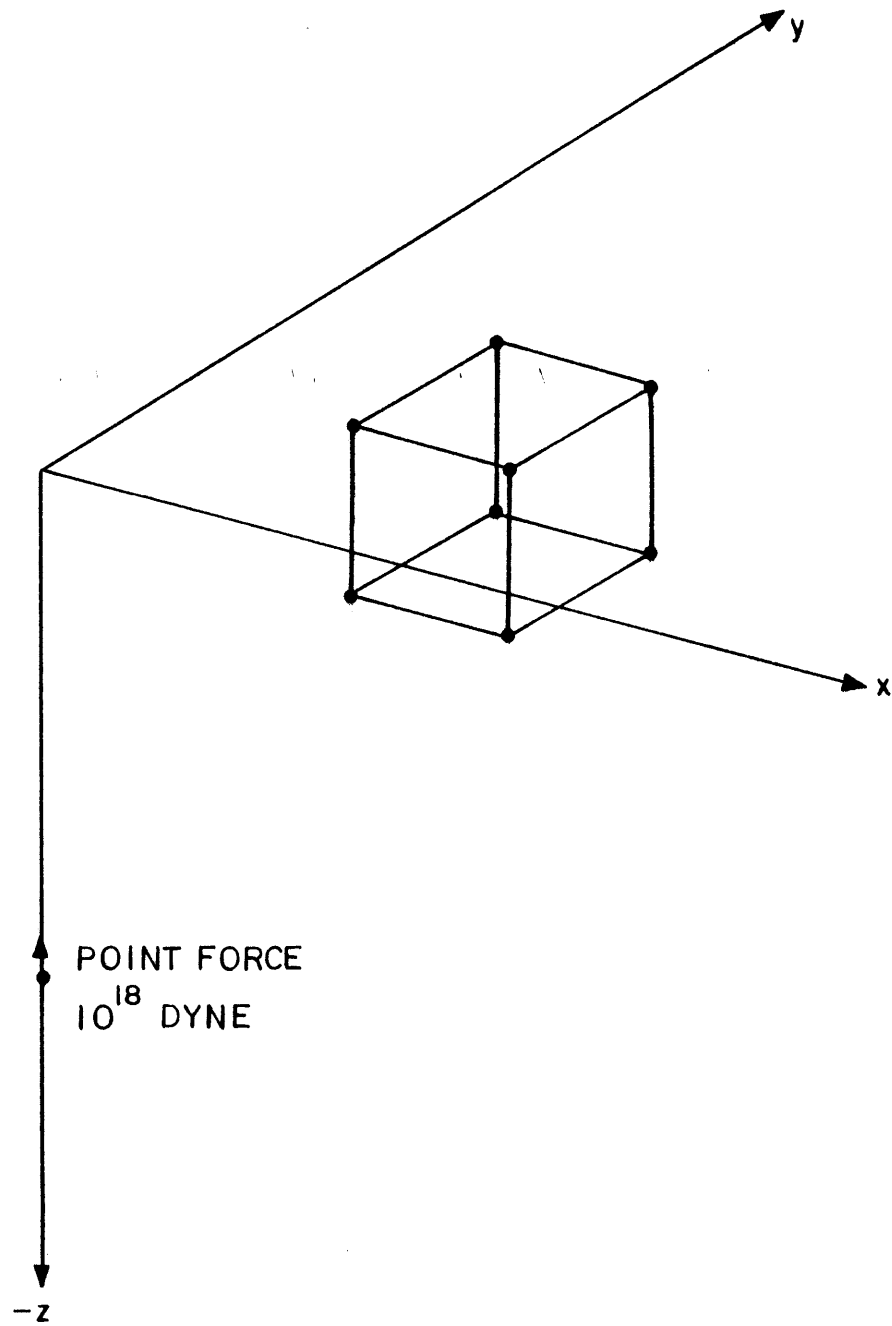


Fig. 8 Model for artificial data



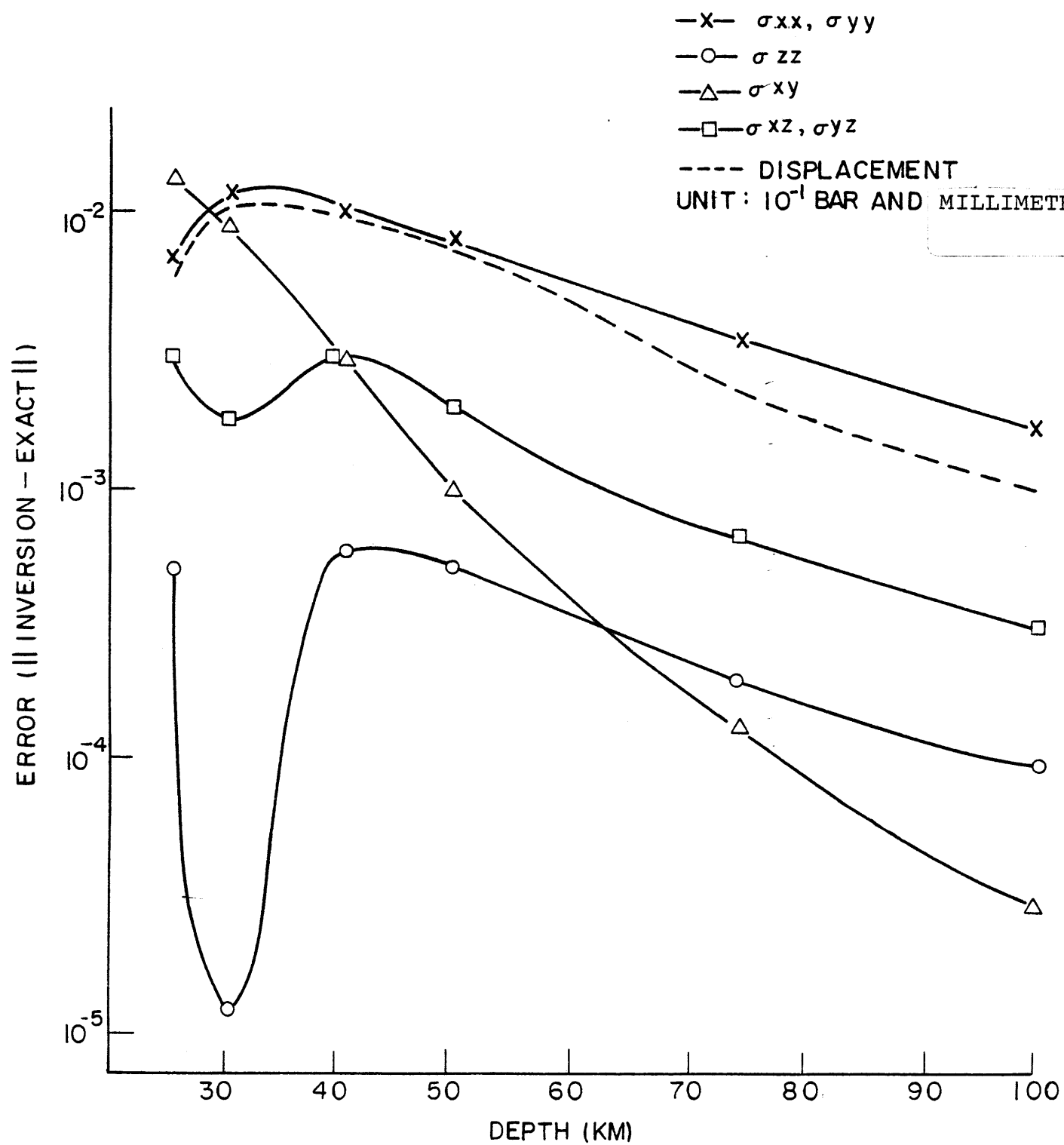


Fig. 9 Error in inversion - 10 x 10 x 10 km block

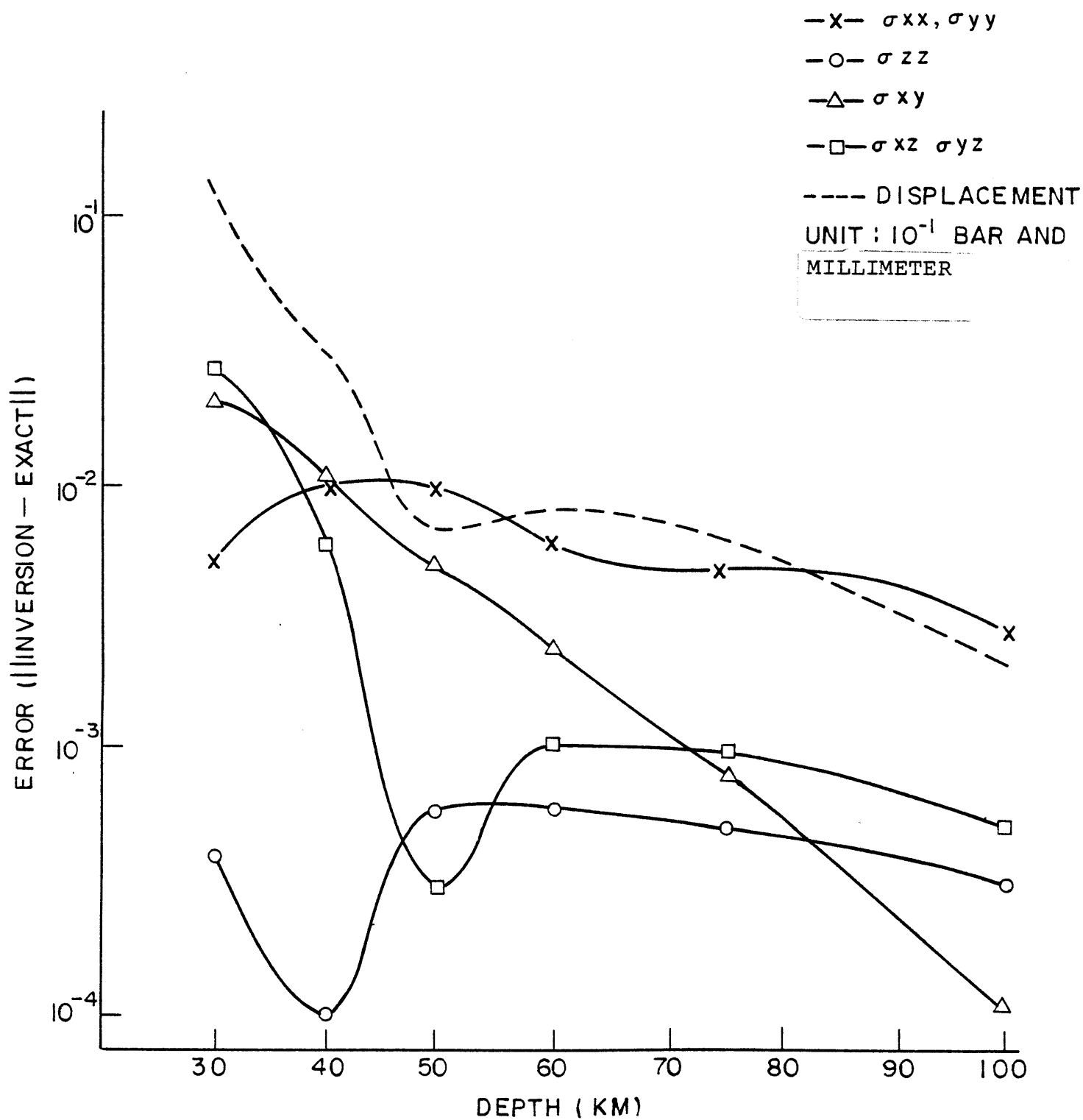


Fig. 10 Error in inversion — 20 x 20 x 20 KM block

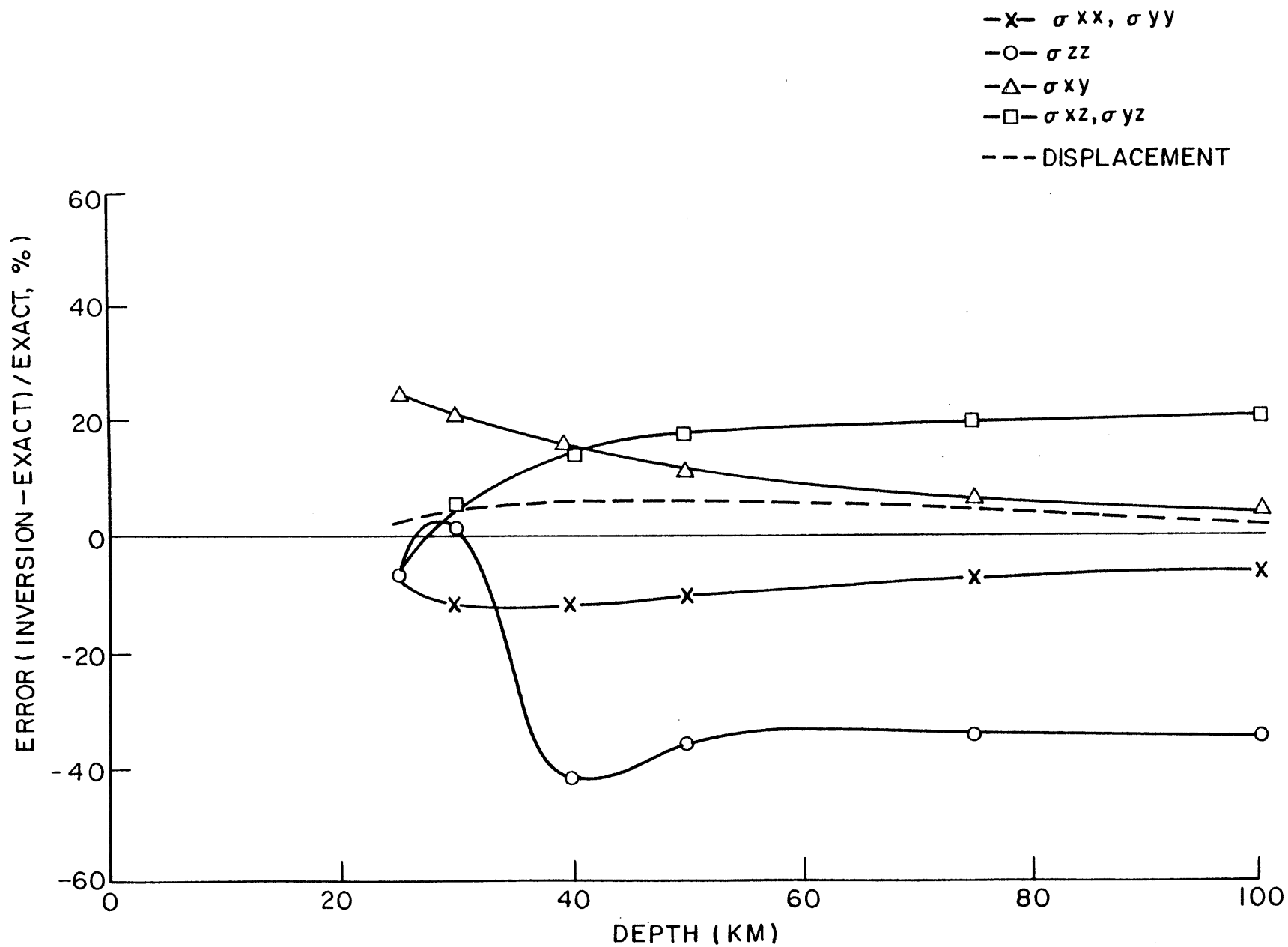


Fig. II Error in the inversion - 10 x 10 x 10 km block

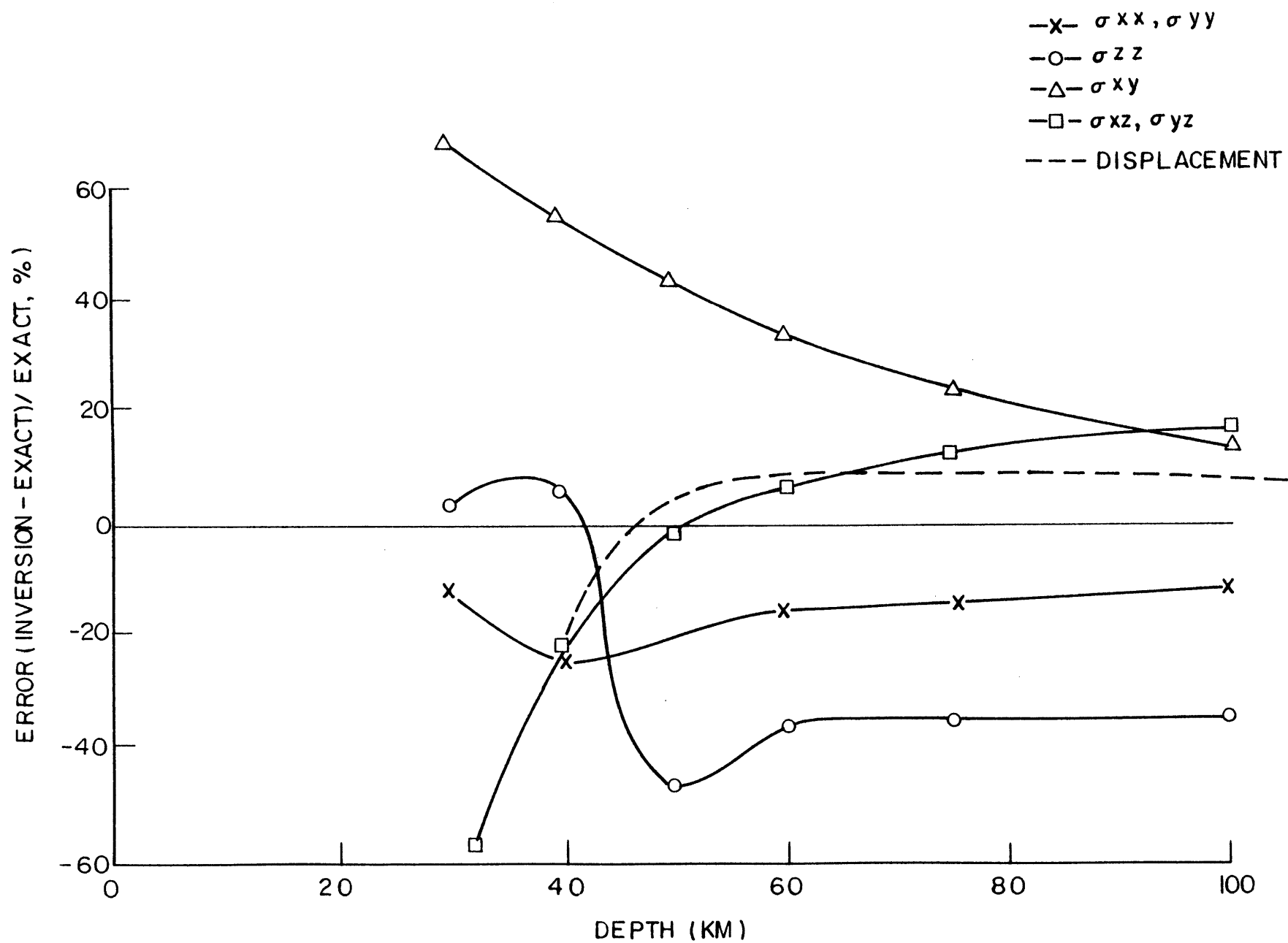


Fig. 12 Error in the inversion – 20 x 20 x 20 KM block

## Chapter 4. PALMDALE UPLIFT

### 1. Levelling Data

The anomalous uplift in Southern California, so-called Palmdale bulge was first discovered in 1975 and reported by Castle et al., in 1976 (Castle et al., 1976). The pattern of uplift is shown in Fig. 13. This first survey was mainly based on the levelling data obtained between 1959 and 1968, and large scale high precision levelling survey was conducted since then. The result of these continued surveys revealed that the bulge was wider than estimated at first and also greater in displacement (Castle, 1978). These surveys also revealed that part of initial uplift subsided since 1974 although the space-time history of this subsidence or "downwarp" is quite uncertain. These results are shown in Fig. 14.

According to Castle (Castle, 1978), the whole episode of uplift occurred as follows:

(1) The uplift began near the intersection of Garlock fault and San Andreas fault in late 1959 and spread eastward. which is confirmed by the continuous levelling near Palmdale showing that this area uplifted 20 cm during 1959-1962 period. This is shown in Fig. 15. The uplift gradually increased by another 15 cm in the next 10 years.

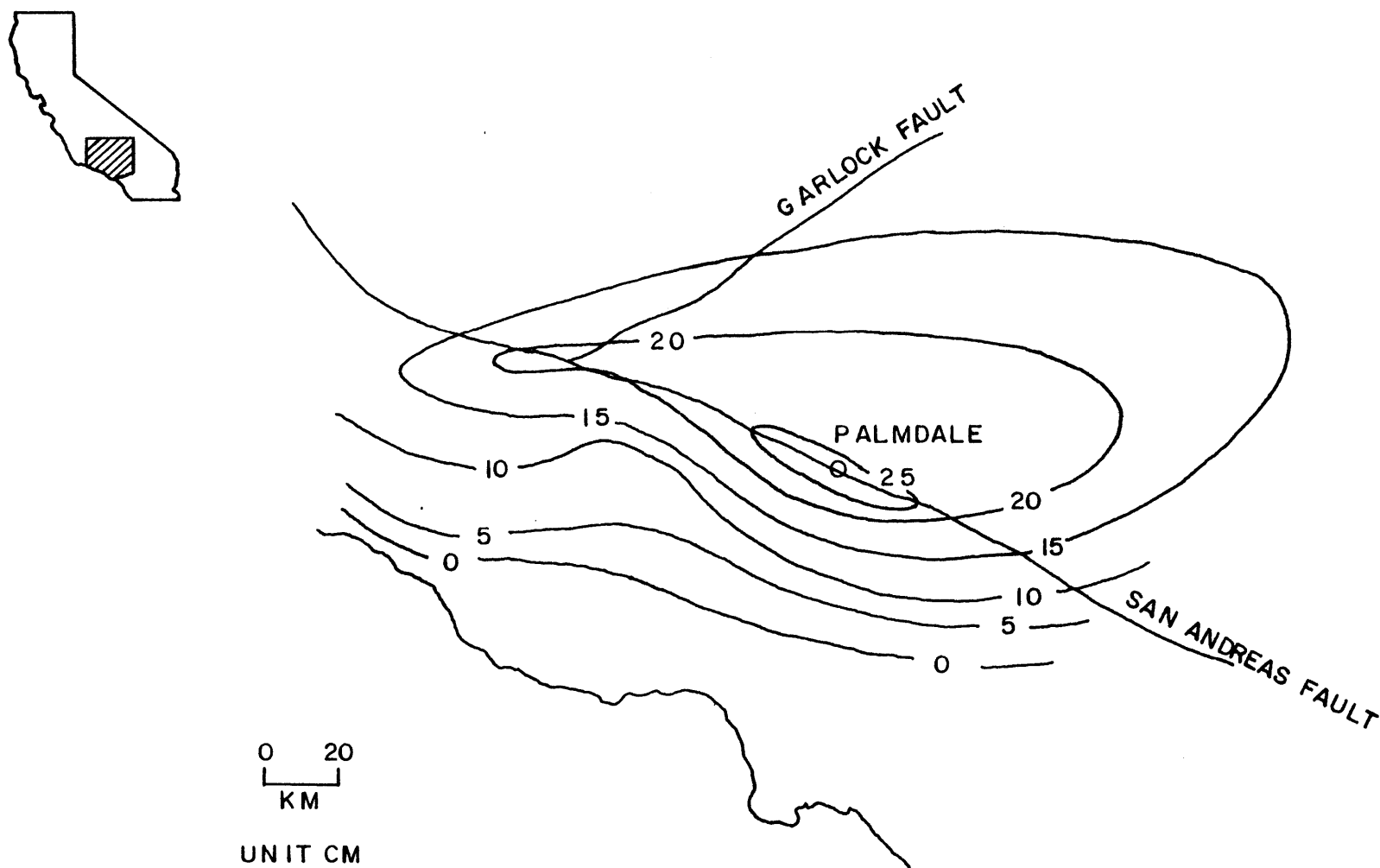


Fig. 13 Palmdale Uplift 1959-1974 (From Castle et al., 1976)

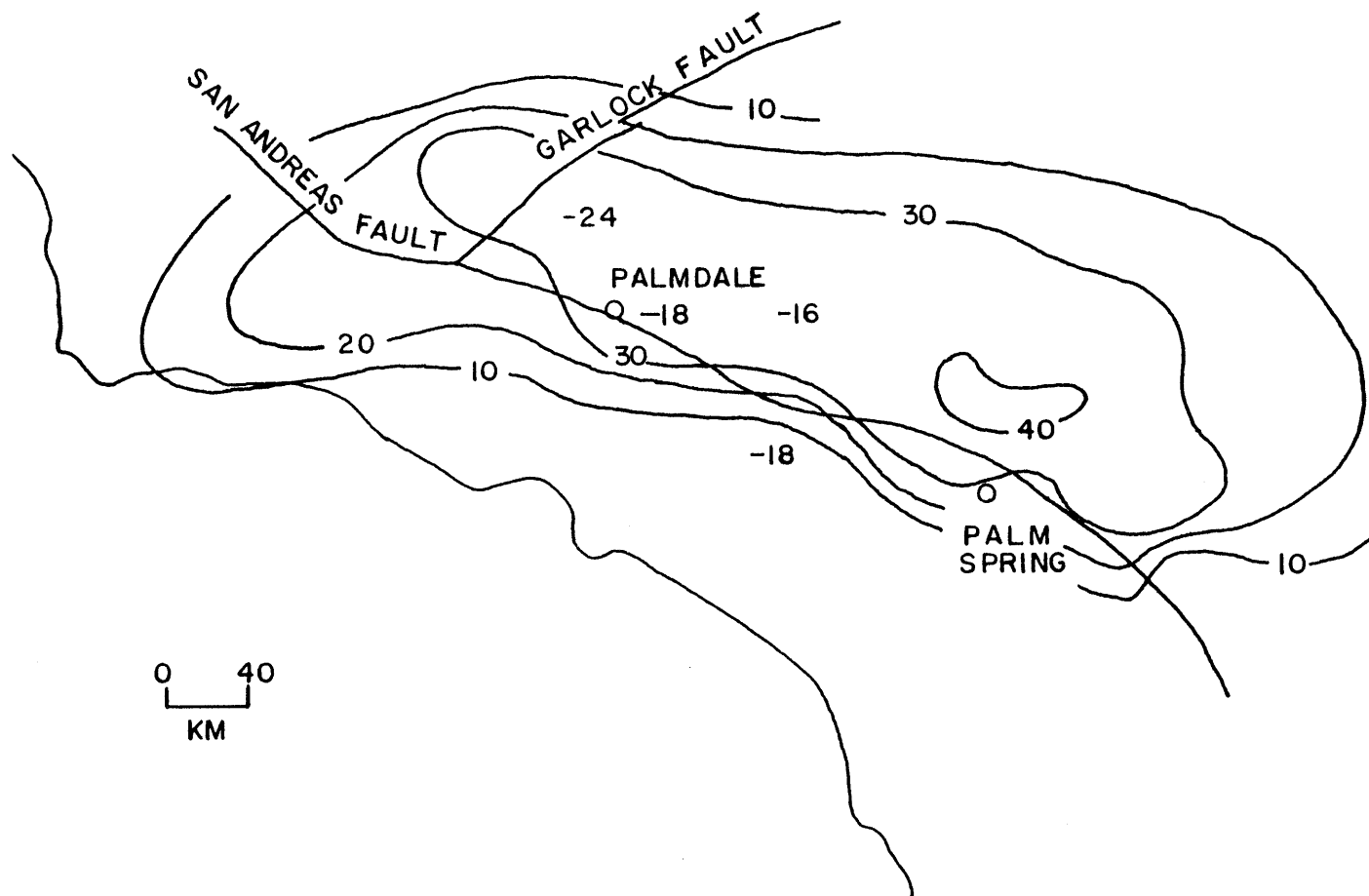


Fig. 14 Palmdale uplift and downwarp 1959-1978 ( From Castle, 1978 )

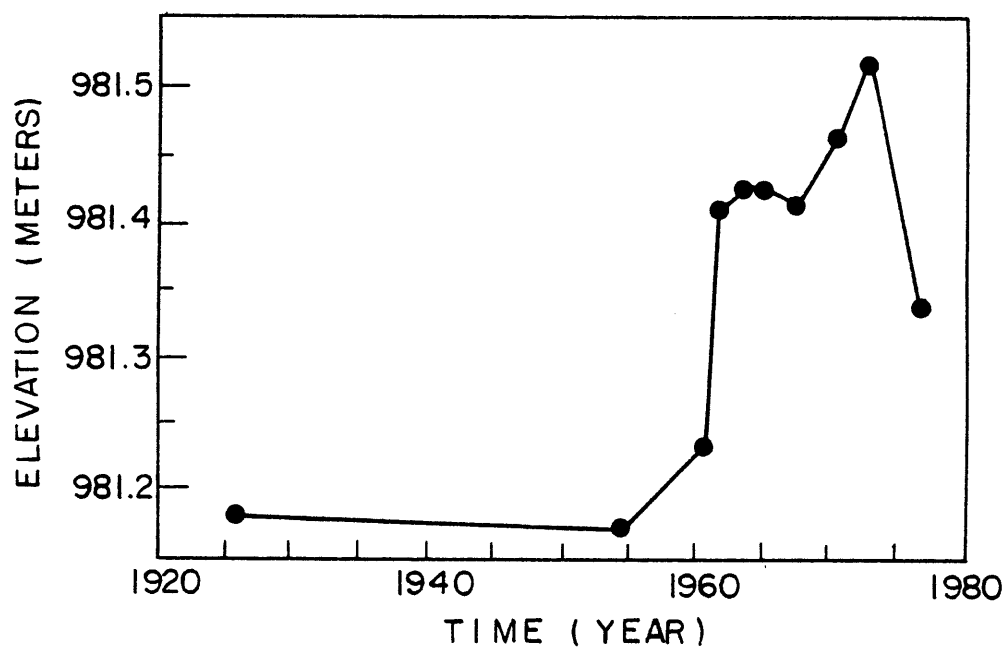


Fig. 15 Levelling data near Palmdale (From Savage and Prescott, 1979)



(2) Between late 1972 and early 1974, the area of uplift expanded to the southeast, where a maximum uplift of 45 cm occurred near Yucca Valley.

(3) Between late 1974 and late 1976, much of the uplift subsided. This downwarp reached 18 cm near Palmdale, 16 cm near Cajon, and 24 cm in Mojave.

## 2. Triangulation Data

Since Reid's suggestion on monitoring a strain accumulation as an earthquake precursor (Reid, 1910), extensive triangulation survey has been conducted in California by many organizations (Savage et al., 1973, Thatcher, 1976). These data show that the general trend of horizontal strain accumulation near Big Bend is  $0.3 \sim 0.4$  microstrain per year of contraction in NS direction and  $0.0 \sim 0.1$  microstrain per year of extension in EW direction. This pattern of strain accumulation is consistent with the regional stress expected here from plate tectonics (Atwater and Molnar, 1973).

After the discovery of Palmdale Bulge, interest in the relationship between horizontal strain accumulation and uplift during the uplift period is increased and many data were re-analyzed.

Using triangulation network near Big Bend (Fig. 16), Thatcher discussed that the direction of strain axes were significantly different from the long term regional trend during the uplift period 1959-1963. He also suggested that the compressional axes are perpendicular to the contour

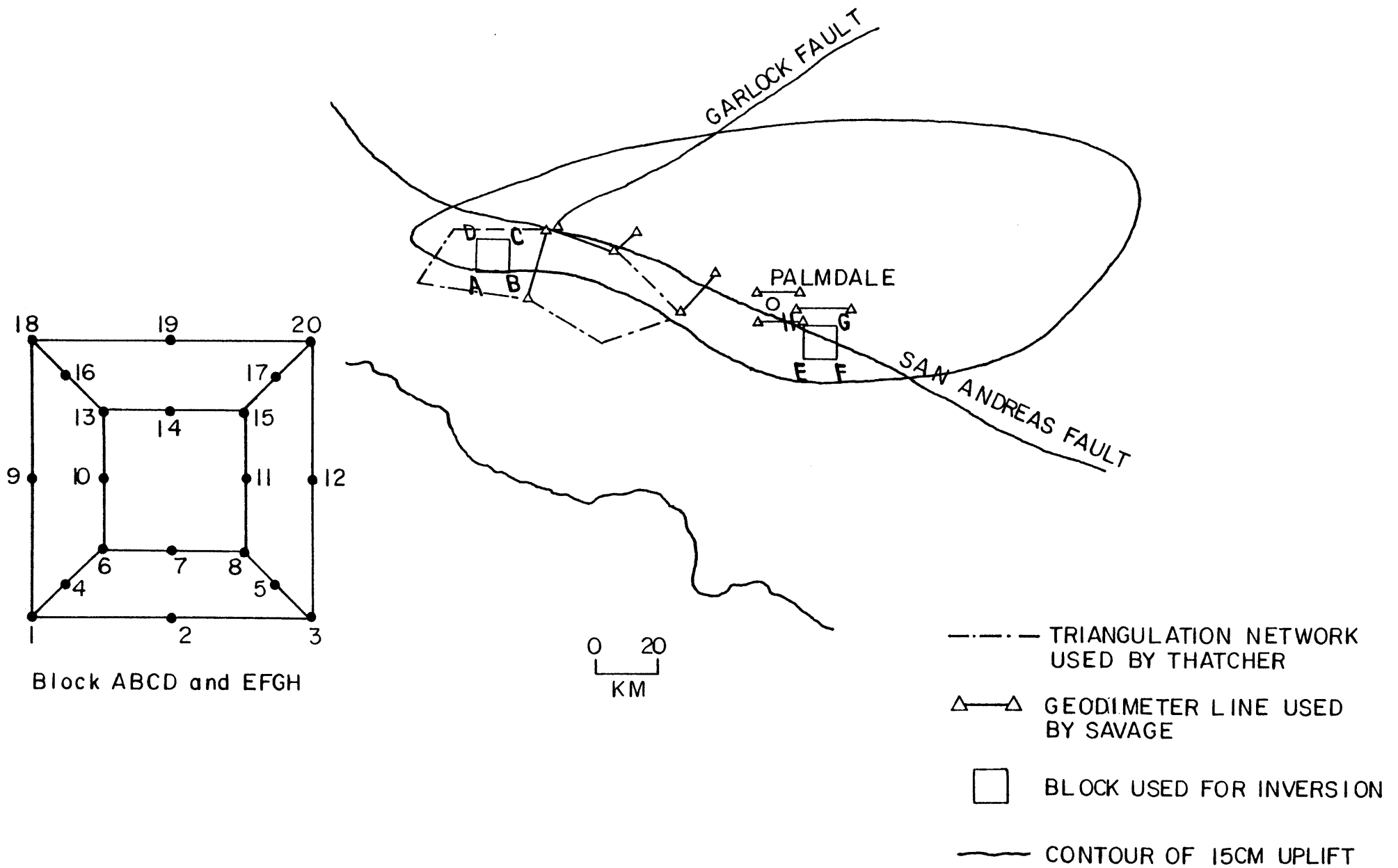


Fig. 16 Geodimeter network and block used for inversion

of uplift almost everywhere (Thatcher, 1976). But his data were based on Frank's method that uses only the change of angle in a triangulation network and therefore we can include any amount of dilatation on his data. As Savage and Prescott stated, this fact makes his result quite model dependent (Savage and Prescott, 1979).

Savage et al. analyzed geodometer data along the San Andreas fault (Fig. 16) and revealed no special change during the period 1950-1972 (Savage et al., 1973). The obtained data shows 0.4 microstrain per year of contraction in N13°E direction and 0.13 microstrain per year of extension in the direction of N77°W near the intersection of Garlock fault and San Andreas fault. Near Palmdale, they observed 0.35 microstrain per year of contraction in the direction of N7°E and 0.07 microstrain per year of extension in the direction of N83°W. These results are based on the change of the length of lines and are therefore not model dependent. Savage et al. also analyzed the strain rate near Palmdale in the period of 1972 to 1978 and revealed that the strain accumulations are 0.3 microstrain per year in NS direction and no strain accumulation in EW direction (Savage et al., 1978).

### 3. Geomagnetic Anomaly

Geomagnetic survey using high precision proton magnetometer were conducted along the 30 km segment of San Andreas fault between Palmdale and San Bernardino in the period 1973-1978 (Johnston et al., 1979). The location

is shown in Fig. 17. The observed anomalous changes were maximum of 10 gamma near Cajon and 5 gamma near Palmdale. These changes are shown in Fig. 17. The increase of magnetic field occurred between the period 1974-1976, which corresponds to the period of partial downwarp of Palmdale bulge. The peak of change was reached in May 1976.

Johnston et al., (1979) discussed that this anomalous change of geomagnetic field can be attributed to the local stress change at the depth less than 10 km. According to his discussion, 5 bar of stress change is sufficient to cause 5 gamma change on the surface if magnetization is  $10^{-2}$  e.m.u. and 50 bar is needed for 5 gamma change if magnetization is  $10^{-3}$  e.m.u.  $10^{-2}$  e.m.u. of magnetization is probably the upper bound in this region (Johnston, 1978).

#### 4. Seismological Data

There is a quite extensive catalog of earthquakes in this region during the period 1931 to 1972 (Heilman et al., 1973) and during the period 1972 to 1974 (Friedman et al., 1976) that show the relative quietness in this region during the period of uplift 1959-1974.

McNally et al. (1978) presented data on an earthquake swarm occurred between late 1976 and late 1977. According to them, in November 1976 an increase in the number of small earthquakes with local magnietuce ( $M_L$ ) 2.0 to 3.0 began in Palmdale area, near Juniper Hills to the southeast and Lake Hughes to the northwest. In the year that followed, 1 November 1976 to 1 November 1977, the

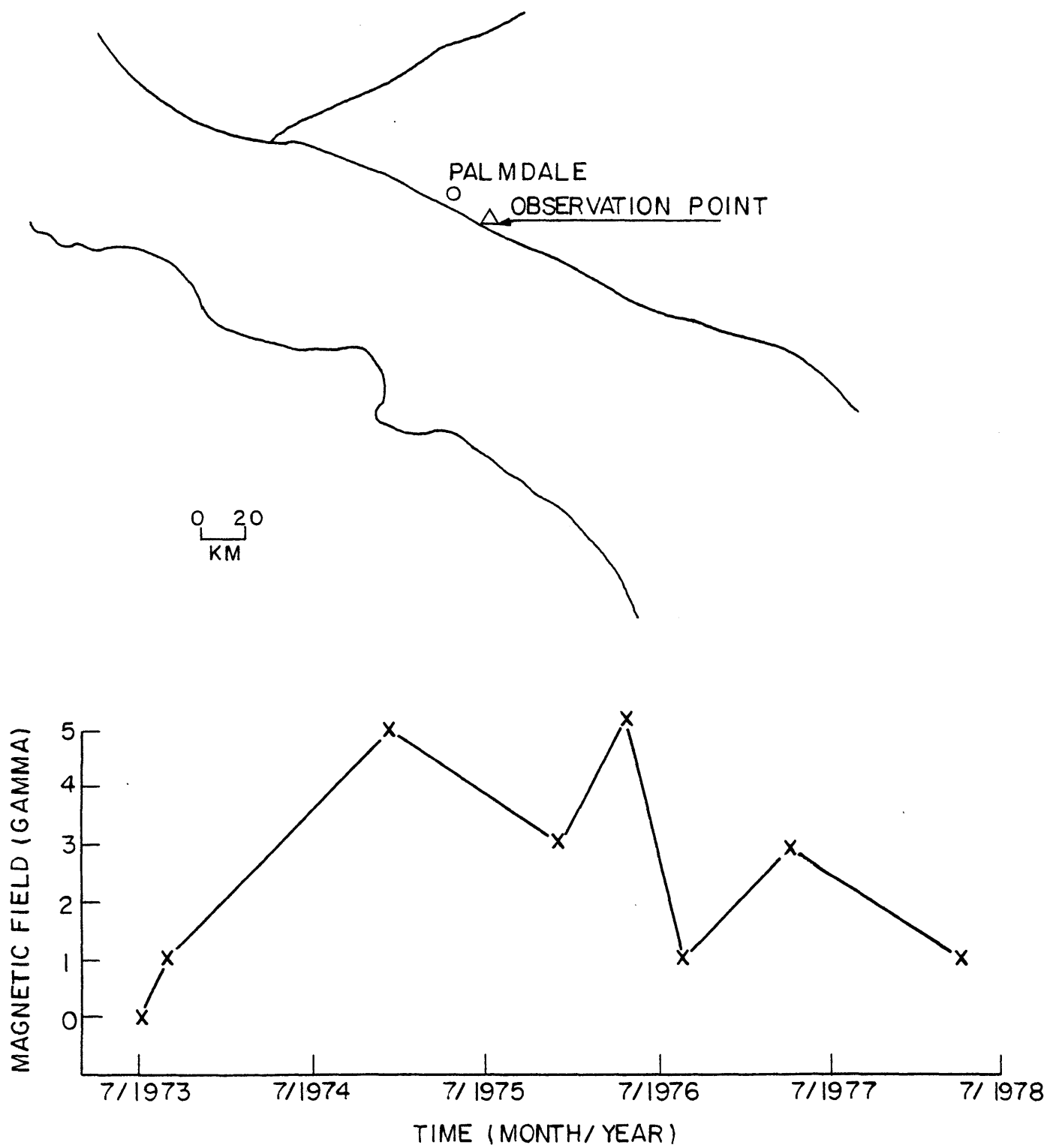


Fig. 17 Geomagnetic data near Palmdale (From Johnston et al. 1979)

number of earthquakes with  $M_L \sim 2.0$  was more than an order of magnitude greater than the long term average for these two areas.

Most earthquakes in the 1976-1977 period are clustered in a small volume with linear dimensions of 3 km (maximum at the depth of 8 km. The fault plane solutions for the largest four earthquakes occurred in the Juniper Hills region in 1976-1977 are shown in Fig. 18, together with the epicenters of these events.

These figures of fault plane solutions show that the axis of maximum compression rotates with time in the clockwise direction from a horizontal northwest-southeast orientation to a horizontal north-south orientation which is more consistent with the regional stress field.

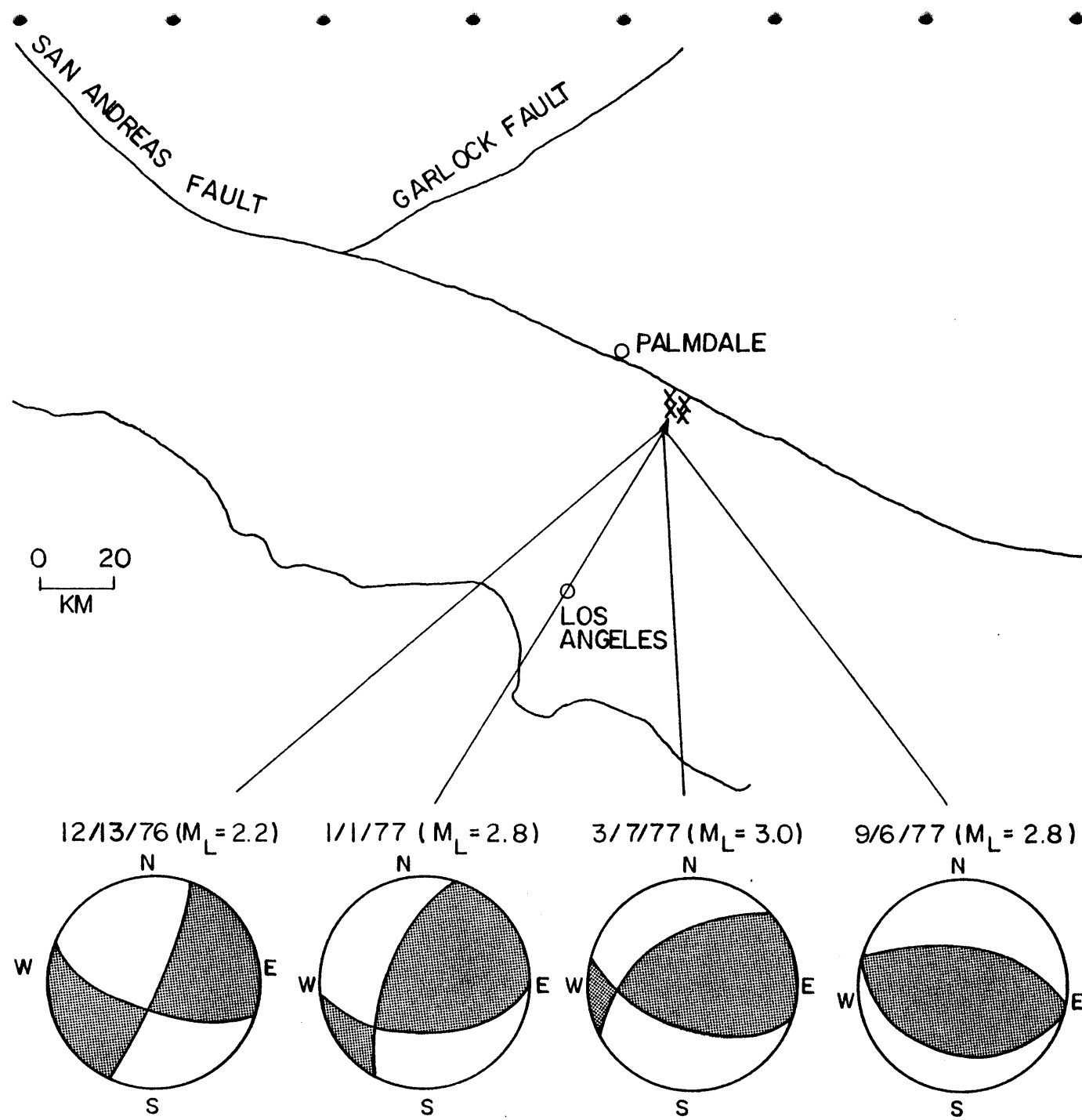


Fig. 18 Earthquake Swarm near Palmdale (From McNally et al. 1978)

## Chapter 5. INVERSION OF PALMDALE DATA

### 1. Data Input

In order to apply our inversion scheme to Palmdale Bulge, the displacement at nodes on the surface are calculated using Savage's data for Block ABCD and Block EFGH shown in Fig. 16. Horizontal displacements are calculated assuming a constant strain rate uniform within each block. One nodal point on the surface was arbitrarily fixed. Vertical displacements are read from a contour map given by Castle et al., (Castle et al., 1976). Rigidity  $3 \times 10^{11}$  dyne/cm<sup>2</sup> and Poisson ratio 0.25 are assumed. It is assumed that through the period of whole episode of uplift and downwarp, the shape of contour does not change, i.e. the distribution of contour line at 1974 (Fig. 13), is preserved. Data used for inversion for the uplift 1959-1962, downwarp 1974-1977 for Block ABCD and Block EFGH are shown in Table 3. Since the temporal distribution of downwarp is not clear, the whole downwarp is assumed to have taken place in one year.

### 2. Result of Inversion

With these three-component displacements data as input to our inversion scheme, we obtained the stress inside the block. The result is shown for the center of each block at depths 3.75 km, 6.5 km and 8.75 km in Fig. 19 through 21, where the principal axes are shown using the Schmidt net (lower hemisphere).



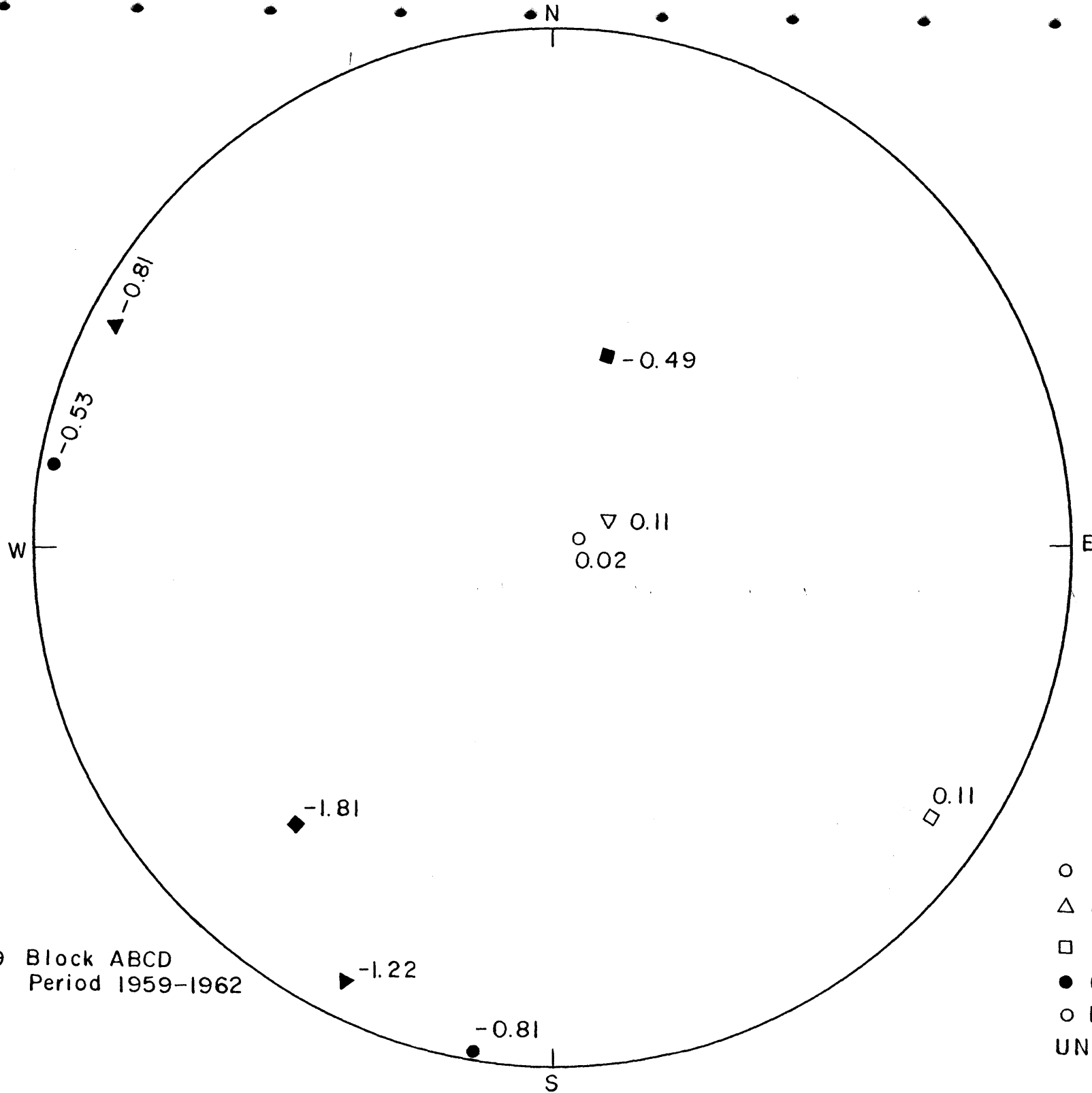


Fig. 19 Block ABCD  
Period 1959-1962

○ DEPTH 3.75 KM  
 △ DEPTH 6.5 KM  
 ◻ DEPTH 8.75 KM  
 ● COMPRESSION  
 ○ EXTENSION  
 UNIT: BAR

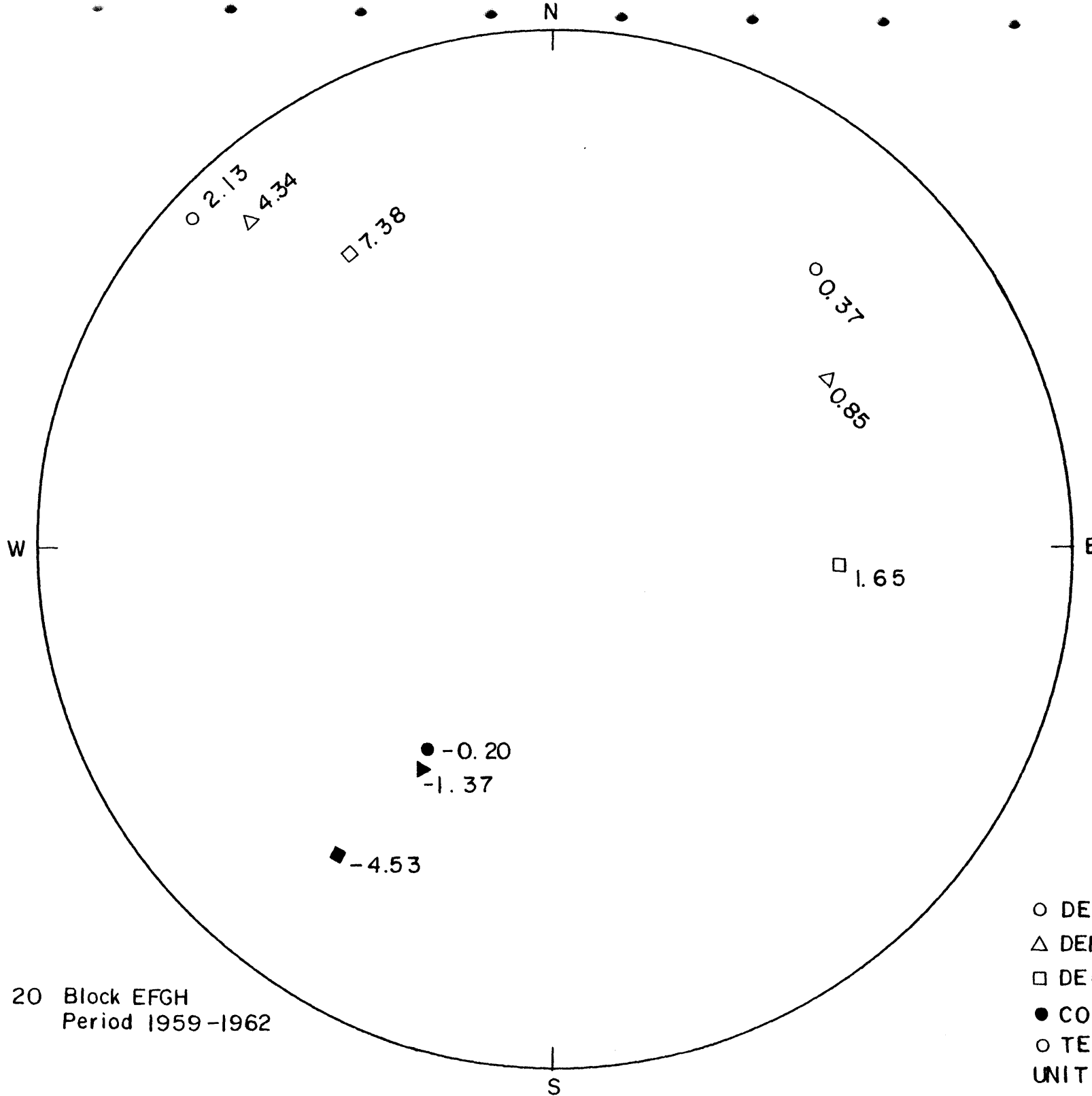


Fig. 20 Block EFGH  
Period 1959-1962

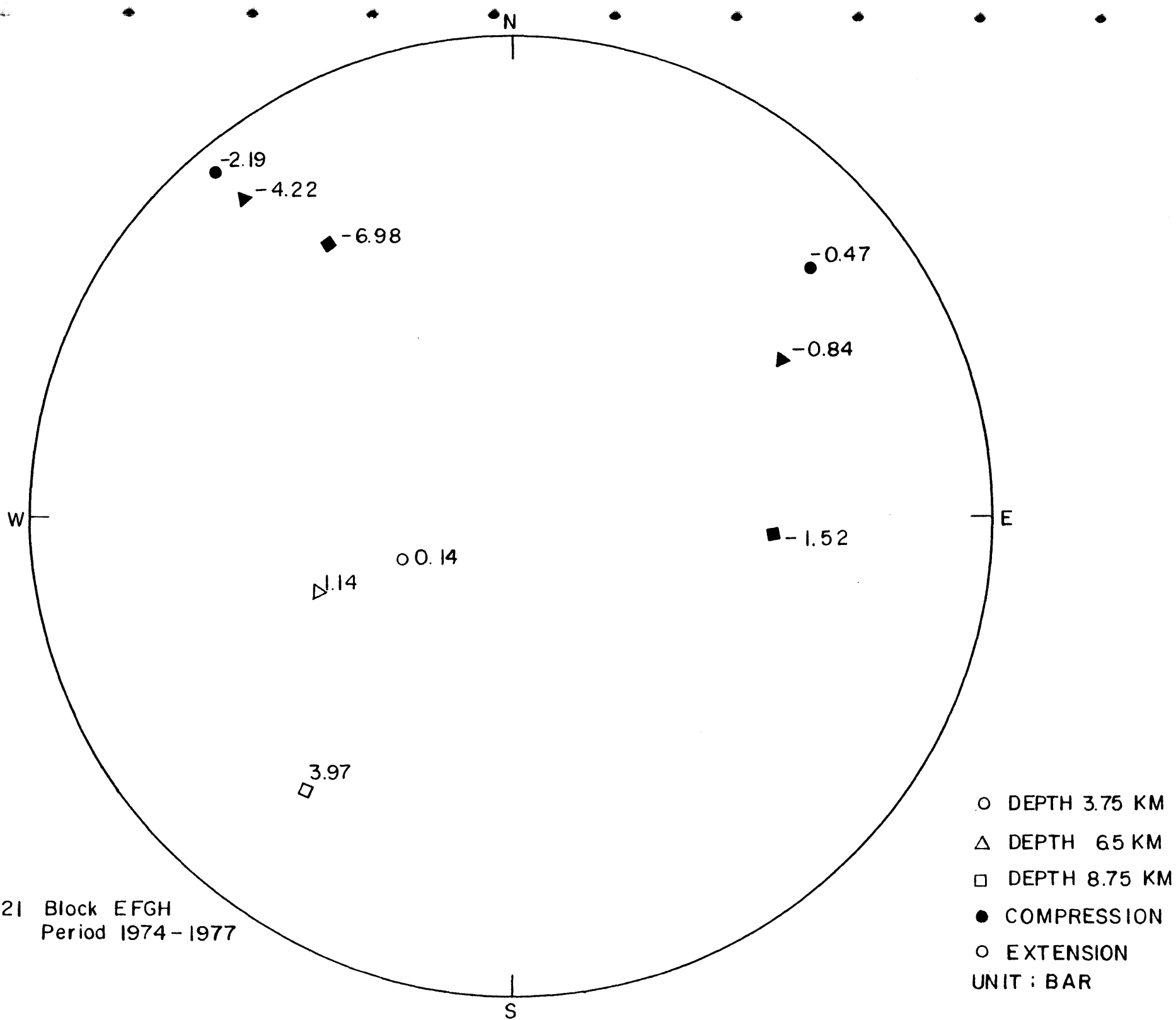


Fig. 21 Block EFGH  
Period 1974 - 1977

The magnitude of principal stress is indicated by numeral in bar in these figures.

Since there is no direct evidence for downwarp in block ABCD, inversion was applied only to the uplift stage for this block. The result from Block ABCD shows that during the period of the uplift at a depth of 3.75 Km, the maximum compressional axis is oriented in the direction of  $N9^{\circ}E$  with magnitude 0.8 bar.

The results from Block EFGH during the period 1959-1962, i.e., the period of uplift is quite interesting because they are showing dominance of tensional stress in that region during uplift. The direction of maximum tension is almost horizontal in the direction of  $N48^{\circ}W$  with the magnitude 2.1 bar at a depth of 3.75 Km. Inversion for the same block during the period 1974-1977, i.e. the period of downwarp gives the horizontal compression axis in the direction of  $N40^{\circ}W$  with magnitude 2.2 bar at a depth of 3.75 Km. This compressional stress increases with depth to 4.2 bar at 6.5 Km and 7.0 bar at 8.75 Km depth without changing directions so much.

Although we don't know exact spatio-temporal distributions of downwarp and we had to make assumptions about the distribution of displacements, the obtained state of stress at depth seems quite compatible with the data obtained by seismological and geomagnetic observations mentioned earlier.

The direction and sense of principle axis during the downwarp period are consistent with the fault plane solutions obtained for earthquakes in the early period of the swarm before the clockwise rotation of axis started.

The magnitude of stress obtained for Block EFGH during the downwarp by this inversion ranges from 2.1 bar at 3.75 Km depth to 7.0 bar at 8.75 Km depth. This result is again compatible with the estimate of Johnston (Johnston 1977, Johnston et al., 1979) for a stress change to account for the observed geomagnetic change of 5 gamma during the period 1974-1978. If the magnetization is  $10^{-2}$  e.m.u., a stress change of 5 bars will produce the observed change in magnetic field.

The state of incremental stress at depth was horizontal tension during the uplift period 1969-1974 and horizontal compression during the downwarp period 1974-1976. The incremental stress before the uplift period was probably horizontal compression but with the rate an order of magnitude smaller than that during the downwarp period. The sudden increase of seismicity during the downwarp period may be due to accelerated horizontal compression.

### 3. Discussion

Mogi (1962) made laboratory fracture experiments on numerous rock-specimen for the purpose of determining the probability  $\lambda(t)$  of fracture occurrence per unit time at time  $t$  measured after the application of a constant stress. He obtained the following formula for  $\lambda(t)$  in the case of granite specimen under bending,

$$\lambda(t) = Ae^{BS} \quad (41)$$

where  $\lambda(t)$  is called "hazard rate" and  $S$  denotes stress. The two constants  $A$ ,  $B$  for the bending experiments of granite-specimen were determined as  $A = 2.0 \times 10^{-12}/\text{year}$  and  $B = 0.37 \text{ cm}^2/\text{Kg}$  (Mogi, 1962).

Hagiwara (1974) modified the above equation to give the hazard rate in terms of ultimate strain which is determined for actual earthquakes statistically using the data on land deformations associated with them (Rikitake, 1974). The equation is

$$\lambda(\epsilon) = \frac{A}{\dot{\epsilon}} e^{BE\epsilon} \quad (42)$$

where  $\lambda(\epsilon)\Delta\epsilon$  is the conditional probability of fracture occurrence in a strain interval  $\epsilon$  and  $\epsilon + \Delta\epsilon$ ,  $\dot{\epsilon}$  is strain rate, and  $E$  is Young's modula. The associated reliability function, or the probability with which the ultimate strain exceed  $\epsilon$ , can be written as

$$R(\epsilon) = \exp \left\{ - \frac{A}{BE \dot{\epsilon}} \exp(BE \epsilon - 1) \right\}. \quad (43)$$

This equation was used to derive the two constants A, B from the statistical distribution of actual ultimate strain assuming Gaussian distribution for the ultimate strain. Using the mean value of ultimate strain as  $5.3 \times 10^{-5}$  with the standard deviation of  $3.3 \times 10^{-5}$  as shown by Rikitake (1974), Young's modulus as  $2.0 \times 10^{11}$  dyne/cm<sup>2</sup> and constant strain rate as  $5.0 \times 10^{-7}$ /year, the values  $A = 0.99 \times 10^{-3}$ /year and  $B = 0.3$  cm<sup>2</sup>/Kg were obtained. It is noticed that if the ultimate strain obtained from the San Francisco earthquake in 1906 is also used in the above calculation, we obtain the values  $A = 1.4 \times 10^{-3}$ /year and  $B = 0.19$  cm<sup>2</sup>/Kg. These variations of constants A, B indicate, together with Scholtz's fracture experiments under the compression test of quartz specimen (Scholtz, 1972) which gave smaller values for B in two orders of magnitude, that these constants may vary significantly for different rocks, different stress conditions and different tectonic history of seismic regions.

Denoting two consecutive periods during stress build-up in an area as period I and period II, we have from eq. (41)

$$\lambda_I = Ae^{B(\sigma_0 + \sigma_I)}$$

$$\lambda_{II} = Ae^{B(\sigma_0 + \sigma_I + \sigma_{II})} \quad (43)$$

where  $\lambda_I$ ,  $\lambda_{II}$  are the probability of fracture occurrences per unit time at the ends of period I and II respectively,  $\sigma_I$  and  $\sigma_{II}$  are incremental stress during the period I and II respectively, and  $\sigma_0$  is the initial stress at the beginning of period I. The value of B can be determined if we know  $\lambda_I$  and  $\lambda_{II}$ , i.e. we have

$$B = \frac{1}{\sigma_{II}} \ln\left(\frac{\lambda_{II}}{\lambda_I}\right). \quad (44)$$

We can estimate the value of A if we know the initial stress  $\sigma_0$ , or alternatively, we can estimate the stress  $\sigma_0$  if the value of A is determined statistically from past data. The equation to be used is

$$\sigma_0 = \frac{1}{B} \ln\left(\frac{\lambda_I}{A}\right) - \sigma_I. \quad (45)$$

According to McNally et al. (1978), the average number of earthquake per year at Juniper Hills before 1953 is 0.6 for events larger than  $M_L \sim 2.0$  and is 15.0 in the period of November 1976 to November 1977. The incremental stress at the focal region is 0.12 bar per year before 1953 and about 7.0 bar in the 1976 to 1977 period as estimated by our inversion. Putting these numbers into eq. (44), the value of B is estimated as  $B \sim 0.47 \text{ cm}^2/\text{Kg}$ . This is



quite comparable to the values obtained by Mogi (1962) and Hagiwara (1974) though it is much larger than the value obtained by Scholtz (1972). This large value of B may be due to the special condition of the particular section of the San Andreas fault.

Though there are observations which indicate a wide variability of the value B (Aki, 1978), the two values obtained from geodetic data, i.e. Hagiwara's statistical data on ultimate strain and our inversion of geodetic data combined with seismic data, and the value obtained from Mogi's experiments are quite comparable and suggests that B may be independent of the scale effect to a certain extent. It may also be possible to extrapolate the value of A obtained from laboratory experiments to the value for actual earthquakes in a large region to estimate the absolute value of probability of the earthquake occurrence. This is a fundamentally important problem for earthquake prediction and will be the subject of our future research.

## Chapter 6. CONCLUSION

Our inversion scheme for determining incremental stress was successfully tested using artificial data generated by a buried point source in a homogeneous half-space.

The scheme was applied to the geodetic data from the Palmdale Bulge, including trilateration, triangulation and levelling data. Most significant conclusion is that the horizontal stress at depth as shallow as a few km can be significantly different from the horizontal stress measured on the surface from trilateration and triangulation alone.

We obtained an encouraging agreement between our estimate of incremental stress and data from geomagnetic and seismic observations for the period of downwarp. During this period, we estimated about 4.2 bars incremental horizontal compression at a depth of 6.5 km. This is compatible with the change of magnetic field by 5 gamma observed by Johnston et al. (Johnston et al., 1979) and increase of seismicity by an order of magnitude observed by McNally et al. (McNally et al., 1978).

These results show that our inversion scheme may be useful for studying various stress-induced earthquake precursor phenomena as a basis for earthquake prediction.

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Table 1. Sampling Points and Weight For Gauss-Legendre Quadrature

Order	$r_i$ (sampling point)	$a_i$ (weight)
1	0.0000000000000000	2.0000000000000000
2	+0.577350269189626	1.0000000000000000
	-0.577350269189626	1.0000000000000000
3	+0.774596669241483	0.5555555555555555
	0.0000000000000000	0.8888888888888888
	-0.774596669241483	0.5555555555555555

Table 2. Values of  $k$ ,  $\ell$ ,  $m$ ,  $n$ 

Case	$k$	$\ell$	$m$	$n$	Total
1	12	12	12	12	48
2	36	36	36	36	144
3	36	36	120	36	228

Case 1. Five 8 nodes element are used (Fig. 3)

Case 2. Five 20 nodes element are used (Fig. 4)

Case 3. Twelve 20 nodes element are used (Fig. 5)

Table 3. Data Input (Unit MM)

## 1. Block ABCD Period 1959-1962

Node	u	v	w
1	.0	.0	114.0000
2	.3734927	-.5479640	117.0000
3	.7469853	-1.095928	120.0000
4	-.4361783D-01	-.6053641	117.0000
5	.5166211	-1.427310	123.6000
6	-.8723567D-01	-1.210728	120.0000
7	.9951066D-01	-1.484710	123.6000
8	.2862570	-1.758692	127.2000
9	-.5479640	-1.873492	130.8000
10	-.3612177	-2.147474	130.2000
11	.1227499D-01	-2.695438	136.8000
12	.1990213	-2.969420	139.8000
13	-.6351997	-3.084221	140.4000
14	-.4484533	-3.358203	143.4000
15	-.2617070	-3.632185	146.4000
16	-.8655638	-3.415603	144.0000
17	-.3053248	-4.237549	153.0000
18	-1.095928	-3.746985	147.6000
19	-.7224353	-4.294949	153.6000
20	-.3489427	-4.842913	159.6000

## 2. Block EFGH Period 1959-1962

Node	u	v	w
1	.0	.0	152.4000
2	.3217809	.2298258	150.6000
3	.6435618	.4596516	148.8000
4	.1379017	-.3229888	156.0000
5	.6205730	.2174995D-01	154.2000
6	.2758034	-.6459776	159.6000
7	.4366938	-.5310647	159.6000
8	.5975843	-.4161517	159.6000
9	.2298258	-1.521781	185.4000
10	.3907163	-1.406868	186.6000
11	.7124972	-1.177042	180.0000
12	.8733876	-1.062129	172.2000
13	.5056292	-2.167759	213.6000
14	.6665196	-2.052846	207.0000
15	.8274101	-1.937933	200.4000
16	.4826404	-2.605660	216.0000
17	.9653118	-2.260921	198.0000
18	.4596516	-3.043562	218.4000
19	.7814325	-2.813736	207.0000
20	1.103213	-2.583910	195.6000

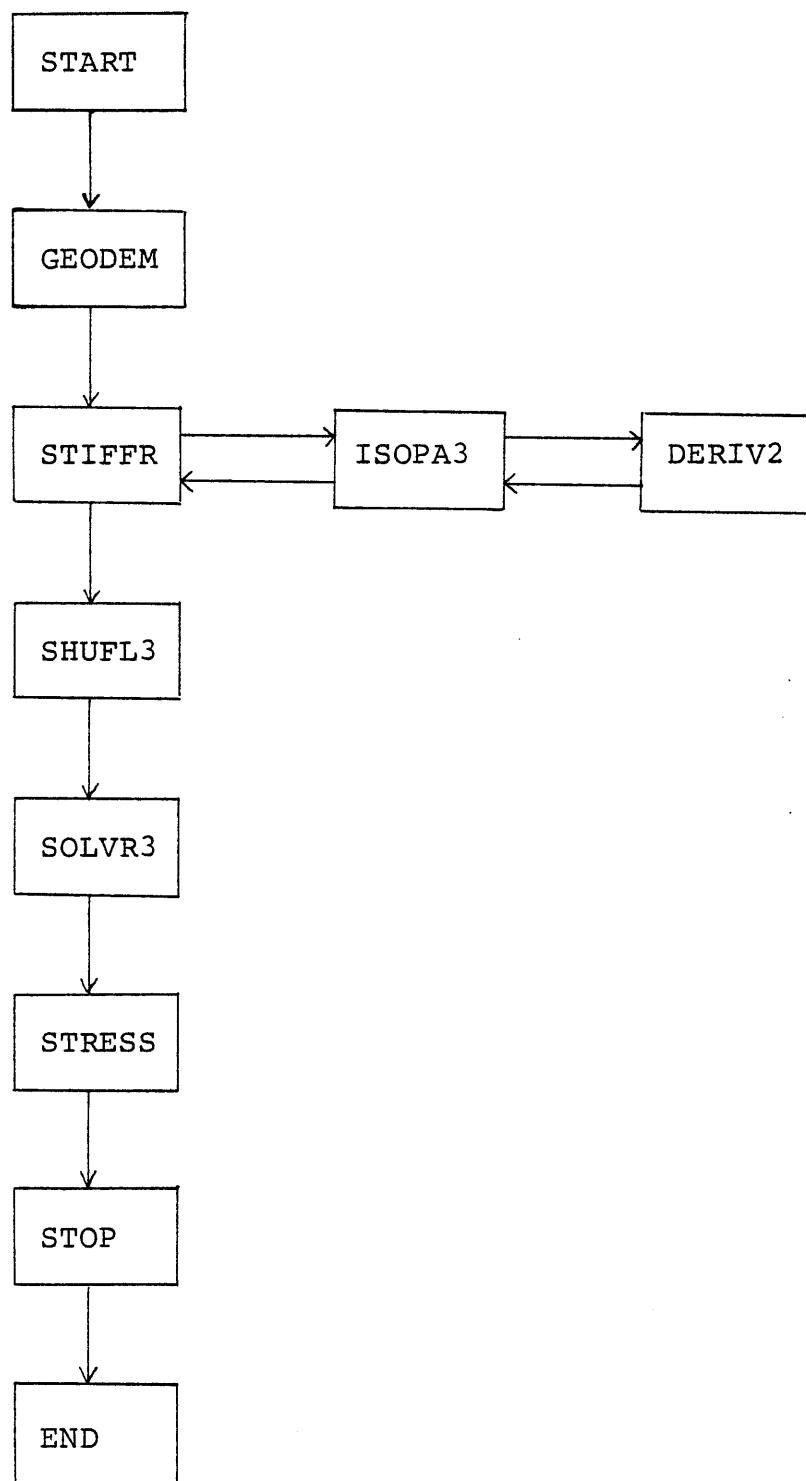
Table 3 (cont'd.)

## 3. Block EFGH Period 1974-1977

Node	u	v	w
1	.0	.0	-139.7000
2	.3000000	.0	-138.0500
3	.6000000	.0	-136.4000
4	.7500000D-01	-.3250000	-143.0000
5	.5250000	-.3250000	-141.3500
6	.1500000	-.6500000	-146.3000
7	.3000000	-.6500000	-146.3000
8	.4500000	-.6500000	-146.3000
9	.0	-1.300000	-169.9500
10	.1500000	-1.300000	-171.0500
11	.4500000	-1.300000	-165.0000
12	.6000000	-1.300000	-157.8500
13	.1500000	-1.950000	-195.8000
14	.3000000	-1.950000	-189.7500
15	.4500000	-1.950000	-183.7000
16	.7500000D-01	-2.275000	-198.0000
17	.5250000	-2.275000	-181.5000
18	.0	-2.600000	-200.2000
19	.3000000	-2.600000	-189.7500
20	.6000000	-2.600000	-179.3000

## Appendix: Code Geodeinverse

Flow Chart.



[illegible]



```

DO 1 I=85,168
1  IROW1(I)=60+I
  DATA ITEMP/1,2,3,9,12,18,19,20,37,38,39,40,41,42,43,44,45,46,47,48
X/
  NUM=MAXCOL/3
  DO 2 I=1,NUM
    IROW2(3*I-2)=3*ITEMP(I)-2
    IROW2(3*I-1)=3*ITEMP(I)-1
2    IROW2(3*I)=3*ITEMP(I)
    MAXDEG=MAXND*3
    MAX1=MAXDEG*2-(MAXCOL+MAXROW)
    MAX2=MAXCOL+MAXROW
    M1=MAXDEG
    M2=MAX1
    M3=M1+1
    MAXID=20*MAXEL
    CALL GEODEM(ITYPE,MAXDEG,MAXND,MAXSND,THETA,SX,SY,SS,
XXCORD,YCORD,ZCORD,XDATA,YDATA,ZDATA,TEMP)
    CALL STIFFR(MAXND,MAXDEG,MGRID,NGRID,LGRID,ELAST,STIFF,XCORD,
XYCORD,ZCORD,NFREE,MAXEL,IDMTX,MAXID,ESTOR)
    CALL SHUFL3(MAXDEG,MAXCOL,MAXROW,STIFF,ICOL1,ICOL2,IROW1,IRO
XW2,GLOAD,TEMP)
    CALL SOLVR3(M1,M2,M3,STIFF,GLOAD,GLOAD2,EPSLON,OMEGA)
    CALL STRESS(MAXDEG,MAXEL,MAXID,MAXCOL,ICOL1,GLOAD2,TEMP,IDMTX,ESTO
XR,GSOL1,ELAST,STOR2)
  STOP
  END

```

```

  SUBROUTINE GEODEM(ITYPE,MAXDEG,MAXND,MAXSND,THETA,SX,SY,SS,
XXCORD,YCORD,ZCORD,XDATA,YDATA,ZDATA,TEMP)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C  SUBROUTINE FOR CALCULATING NECESSARY THREE COMPONENTS OF
C  DISPLACEMENTS ON THE SURFACE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  IMPLICIT REAL*8(A-H,O-Z)
  DIMENSION XCORD(MAXND),YCORD(MAXND),ZCORD(MAXND),TEMP(MAXDEG),XDAT
XA(MAXSND),YDATA(MAXSND),ZDATA(MAXSND),ID(36),ZDATA2(100),ID2(8)
  PI=3.14159265
  DATA ID/1,2,3,3,12,20,20,19,18,18,9,1,1,4,6,3,5,8,20,17,15,18,16,1
X3,6,7,8,8,11,15,15,14,13,13,10,6/
  DATA ID2/1,3,6,8,13,15,18,20/
C  SS REPRESENTS TENSOR SHEAR
  SS=2.*SS
  THETA2=THETA*PI/180.
  C1=DCOS(THETA2)
  C2=DSIN(THETA2)
  DO 30 I=1,MAXSND
    X=XCORD(I)-XCORD(1)

```

```

      Y=YCORD(I)-YCORD(1)
      DX=C1*X-C2*Y
      DY=C2*X+C1*Y
      IF(ITYPE) 10,10,20
10    DELX=SX*DX
      DELY=SY*DY
      XDATA(I)=DELX*C1+DELY*C2
      YDATA(I)=-DELX*C2+DELY*C1
      GOTO 30
20    DELX=0.
      DELY=-SS*DX
      XDATA(I)=DELY*C2
      YDATA(I)=DELY*C1
30    CONTINUE
      DO 31 I=1,8
31    ZDATA2(ID2(I))=ZDATA(I)
      DO 32 I=1,34,3
      I1=ID(I)
      I2=ID(I+1)
      I3=ID(I+2)
32    ZDATA2(I2)=.5*(ZDATA2(I1)+ZDATA2(I3))
      DO 40 I=1,MAXDEG
40    TEMP(I)=0.
      DO 41 I=1,MAXSND
      TEMP(3*I-2)=XDATA(I)
      TEMP(3*I-1)=YDATA(I)
41    TEMP(3*I)=ZDATA2(I)
      WRITE(6,1000)
      DO 42 I=1,MAXSND
42    WRITE(6,1001) I,TEMP(3*I-2),TEMP(3*I-1),TEMP(3*I)
1000  FORMAT(1H0,'NODE',6X,'X',15X,'Y',15X,'Z',/)
1001  FORMAT(I3,3X,3G15.7)
      RETURN
      END

```

```

      SUBROUTINE STIFFR(MAXND,MAXDEG,MGRID,NGRID,LGRID,ELAST,DUMMY,XCORD
      X,YCORD,ZCORD,NFREE,MAXEL,IDMTX,MAXID,ESTOR)
      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      C                                                                 C
      C      SUBROUTINE FOR GENERATING STIFFNESS MATRIX BASED ON 20-NODES  C
      C      PARABOLIC HEXAHEDRAL ELEMENT                                C
      C                                                                 C
      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DUMMY(MAXDEG,MAXDEG),XCORD(MAXND),YCORD(MAXND),ZCORD(MAX
      XND),ELAST(6,6),ID(60),X(20),Y(20),Z(20),S(60,60),IDMTX(MAXID),ESTO
      XR(MAXEL,6,60)
      DO 1 I=1,MAXDEG
      DO 1 J=1,MAXDEG
1     DUMMY(I,J)=0.

```



```

S=GS(IS)
T=GT(IT)
CALL DERIV2(DERIV,R,S,T)
C WRITE(6,3001)
C WRITE(6,3000) (DERIV(1,J),DERIV(2,J),DERIV(3,J),J=1,20)
3001 FORMAT(//)
DO 20 I=1,3
DO 20 K=1,20
A(I,1)=A(I,1)+DERIV(I,K)*X(K)
A(I,2)=A(I,2)+DERIV(I,K)*Y(K)
20 A(I,3)=A(I,3)+DERIV(I,K)*Z(K)
DET=A(1,1)*A(2,2)*A(3,3)+A(1,2)*A(2,3)*A(3,1)+A(1,3)*A(3,2)*A(2,1)
X-A(1,3)*A(2,2)*A(3,1)-A(1,2)*A(2,1)*A(3,3)-A(1,1)*A(3,2)*A(2,3)
IF(DET.LE.1.0E-04) GO TO 80
B(1,1)=A(2,2)*A(3,3)-A(2,3)*A(3,2)
B(1,2)=A(3,2)*A(1,3)-A(1,2)*A(3,3)
B(1,3)=A(1,2)*A(2,3)-A(1,3)*A(2,2)
B(2,1)=A(2,3)*A(3,1)-A(2,1)*A(3,3)
B(2,2)=A(1,1)*A(3,3)-A(1,3)*A(3,1)
B(2,3)=A(1,3)*A(2,1)-A(1,1)*A(2,3)
B(3,1)=A(2,1)*A(3,2)-A(2,2)*A(3,1)
B(3,2)=A(1,2)*A(3,1)-A(1,1)*A(3,2)
B(3,3)=A(1,1)*A(2,2)-A(1,2)*A(2,1)
DO 30 I=1,3
DO 30 J=1,3
30 B(I,J)=B(I,J)/DET
DO 40 I=1,3
DO 40 J=1,20
D(I,3*J-2)=DERIV(I,J)
D(I+3,3*J-1)=DERIV(I,J)
40 D(I+6,3*J)=DERIV(I,J)
DO 50 J=1,3
C(1,J)=B(1,J)
C(2,J+3)=B(2,J)
C(3,J+6)=B(3,J)
C(4,J)=B(2,J)
C(4,J+3)=B(1,J)
C(5,J)=B(3,J)
C(5,J+6)=B(1,J)
C(6,J+3)=B(3,J)
50 C(6,J+6)=B(2,J)
CALL MULTD(6,9,60,C,D,E)
CALL MULTD(6,6,60,ELAST,E,CE)
IF(IR-2) 55,51,55
51 IF(IS-2) 55,52,55
52 IF(IT-2) 55,53,55
53 DO 54 I=1,6
DO 54 J=1,60
54 ESTOR(NEL,I,J)=E(I,J)
55 CONTINUE
DO 60 I=1,6
DO 60 J=1,60
60 ET(J,I)=E(I,J)
CALL MULTD(60,6,60,ET,CE,SMATX)

```

```

SUBROUTINE DERIV2 (DERIV,R,S,T)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE FOR   CALCULATING JACOBIAN MATRIX
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION DERIV(3,20)
      DATA NODE/20/
      DERIV(1,1)=(1.+S)*(1.+T)*.125
      DERIV(1,2)=(1.+S)*(1.+T)*(-.125)
      DERIV(1,3)=(1.-S)*(1.+T)*(-.125)
      DERIV(1,4)=(1.-S)*(1.+T)*.125
      DERIV(1,5)=(1.+S)*(1.-T)*.125
      DERIV(1,6)=(1.+S)*(1.-T)*(-.125)
      DERIV(1,7)=(1.-S)*(1.-T)*(-.125)
      DERIV(1,8)=(1.-S)*(1.-T)*.125
      DERIV(2,1)=(1.+R)*(1.+T)*.125
      DERIV(2,2)=(1.-R)*(1.+T)*.125
      DERIV(2,3)=(1.-R)*(1.+T)*(-.125)
      DERIV(2,4)=(1.+R)*(1.+T)*(-.125)
      DERIV(2,5)=(1.+R)*(1.-T)*.125
      DERIV(2,6)=(1.-R)*(1.-T)*.125

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DERIV(2,6)=(1.-R)*(1.-T)*.125
DERIV(2,7)=(1.-R)*(1.-T)*(-.125)
DERIV(2,8)=(1.+R)*(1.-T)*(-.125)
DERIV(3,1)=(1.+R)*(1.+S)*.125
DERIV(3,2)=(1.-R)*(1.+S)*.125
DERIV(3,3)=(1.-R)*(1.-S)*.125
DERIV(3,4)=(1.+R)*(1.-S)*.125
DERIV(3,5)=(1.+R)*(1.+S)*(-.125)
DERIV(3,6)=(1.-R)*(1.+S)*(-.125)
DERIV(3,7)=(1.-R)*(1.-S)*(-.125)
DERIV(3,8)=(1.+R)*(1.-S)*(-.125)
IF(NODE.EQ.8) GOTO 10
DERIV(1,9)=-.5*R*(1.+S)*(1.+T)
DERIV(1,10)=-.25*(1.-S*S)*(1.+T)
DERIV(1,11)=-.5*R*(1.-S)*(1.+T)
DERIV(1,12)=.25*(1.-S*S)*(1.+T)
DERIV(1,13)=-.5*R*(1.+S)*(1.-T)
DERIV(1,14)=-.25*(1.-S*S)*(1.-T)
DERIV(1,15)=-.5*R*(1.-S)*(1.-T)
DERIV(1,16)=.25*(1.-S*S)*(1.-T)
DERIV(1,17)=.25*(1.+S)*(1.-T*T)
DERIV(1,18)=-.25*(1.+S)*(1.-T*T)
DERIV(1,19)=-.25*(1.-S)*(1.-T*T)
DERIV(1,20)=.25*(1.-S)*(1.-T*T)
DERIV(1,1)=DERIV(1,1)-.5*(DERIV(1,9)+DERIV(1,12)+DERIV(1,17))
DERIV(1,2)=DERIV(1,2)-.5*(DERIV(1,9)+DERIV(1,10)+DERIV(1,18))
DERIV(1,3)=DERIV(1,3)-.5*(DERIV(1,10)+DERIV(1,11)+DERIV(1,19))
DERIV(1,4)=DERIV(1,4)-.5*(DERIV(1,11)+DERIV(1,12)+DERIV(1,20))
DERIV(1,5)=DERIV(1,5)-.5*(DERIV(1,16)+DERIV(1,17)+DERIV(1,13))
DERIV(1,6)=DERIV(1,6)-.5*(DERIV(1,13)+DERIV(1,14)+DERIV(1,18))
DERIV(1,7)=DERIV(1,7)-.5*(DERIV(1,14)+DERIV(1,15)+DERIV(1,19))
DERIV(1,8)=DERIV(1,8)-.5*(DERIV(1,15)+DERIV(1,16)+DERIV(1,20))
DERIV(2,9)=.25*(1.-R*R)*(1.+T)
DERIV(2,10)=-.5*(1.-R)*S*(1.+T)
DERIV(2,11)=-.25*(1.-R*R)*(1.+T)
DERIV(2,12)=-.5*(1.+R)*S*(1.+T)
DERIV(2,13)=.25*(1.-R*R)*(1.-T)
DERIV(2,14)=-.5*(1.-R)*S*(1.-T)
DERIV(2,15)=-.25*(1.-R*R)*(1.-T)
DERIV(2,16)=-.5*(1.+R)*S*(1.-T)
DERIV(2,17)=.25*(1.+R)*(1.-T*T)
DERIV(2,18)=.25*(1.-R)*(1.-T*T)
DERIV(2,19)=-.25*(1.-R)*(1.-T*T)
DERIV(2,20)=-.25*(1.+R)*(1.-T*T)
DERIV(2,1)=DERIV(2,1)-.5*(DERIV(2,9)+DERIV(2,12)+DERIV(2,17))
DERIV(2,2)=DERIV(2,2)-.5*(DERIV(2,9)+DERIV(2,10)+DERIV(2,18))
DERIV(2,3)=DERIV(2,3)-.5*(DERIV(2,10)+DERIV(2,11)+DERIV(2,19))
DERIV(2,4)=DERIV(2,4)-.5*(DERIV(2,11)+DERIV(2,12)+DERIV(2,20))
DERIV(2,5)=DERIV(2,5)-.5*(DERIV(2,16)+DERIV(2,17)+DERIV(2,13))
DERIV(2,6)=DERIV(2,6)-.5*(DERIV(2,13)+DERIV(2,14)+DERIV(2,18))
DERIV(2,7)=DERIV(2,7)-.5*(DERIV(2,14)+DERIV(2,15)+DERIV(2,19))
DERIV(2,8)=DERIV(2,8)-.5*(DERIV(2,15)+DERIV(2,16)+DERIV(2,20))
DERIV(3,9)=.25*(1.-R*R)*(1.+S)
DERIV(3,10)=.25*(1.-R)*(1.-S*S)

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DERIV(3,11)=.25*(1.-R*R)*(1.-S)
DERIV(3,12)=.25*(1.+R)*(1.-S*S)
DERIV(3,13)=-.25*(1.-R*R)*(1.+S)
DERIV(3,14)=-.25*(1.-R)*(1.-S*S)
DERIV(3,15)=-.25*(1.-R*R)*(1.-S)
DERIV(3,16)=-.25*(1.+R)*(1.-S*S)
DERIV(3,17)=-.5*(1.+R)*(1.+S)*T
DERIV(3,18)=-.5*(1.-R)*(1.+S)*T
DERIV(3,19)=-.5*(1.-R)*(1.-S)*T
DERIV(3,20)=-.5*(1.+R)*(1.-S)*T
DERIV(3,1)=DERIV(3,1)-.5*(DERIV(3,9)+DERIV(3,12)+DERIV(3,17))
DERIV(3,2)=DERIV(3,2)-.5*(DERIV(3,9)+DERIV(3,10)+DERIV(3,18))
DERIV(3,3)=DERIV(3,3)-.5*(DERIV(3,10)+DERIV(3,11)+DERIV(3,19))
DERIV(3,4)=DERIV(3,4)-.5*(DERIV(3,11)+DERIV(3,12)+DERIV(3,20))
DERIV(3,5)=DERIV(3,5)-.5*(DERIV(3,16)+DERIV(3,17)+DERIV(3,13))
DERIV(3,6)=DERIV(3,6)-.5*(DERIV(3,13)+DERIV(3,14)+DERIV(3,18))
DERIV(3,7)=DERIV(3,7)-.5*(DERIV(3,14)+DERIV(3,15)+DERIV(3,19))
DERIV(3,8)=DERIV(3,8)-.5*(DERIV(3,15)+DERIV(3,16)+DERIV(3,20))
10  RETURN
    END

```

```

SUBROUTINE SHUFL3(MAXDEG,MAXCOL,MAXROW,STIFF,ICOL1,ICOL2,IRCW
X1,IROW2,GLOAD,TEMP)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C SUBROUTINE FOR ARRANGING KNOWN AND UNKNOWNNS C
C C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 JUNK,JUNK2
DIMENSION STIFF(MAXDEG,MAXDEG),GLOAD(MAXDEG),TEMP(MAXDEG),
XICOL1(MAXDEG),ICOL2(MAXDEG),IROW1(MAXDEG),IRCW2(MAXDEG)
DO 11 I=1,MAXDEG
11  GLOAD(I)=0.
DO 12 I=1,MAXDEG
DO 12 J=1,MAXCOL
12  GLOAD(I)=GLOAD(I)-STIFF(I,ICOL1(J))*TEMP(J)
DO 20 I=1,MAXDEG
DO 20 J=1,MAXROW
20  STIFF(I,J)=STIFF(I,J+MAXCOL)
DO 21 I=1,MAXDEG
DO 21 J=1,MAXCOL
STIFF(I,J+MAXROW)=0.
21  IF(IROW2(J).EQ.I) STIFF(I,J+MAXROW)=-1.
RETURN
END

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SUBROUTINE SOLVR3(M1,M2,M3,DUMMY4,GLOAD,GLOAD2,EPSLON,OMEGA)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C      SUBROUTINE FOR SOLVING LINEAR EQUATION                      C
C      BASED ON GAUSS JORDAN ELIMINATION USING PARTIAL PIVOTING   C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8(A-H,O-Z)
      ABS(X)=DABS(X)
      DIMENSION DUMMY4(M1,M3),GLOAD(M1),GLOAD2(M1)
      IMAX=M1
      DO 10 I=1,IMAX
10    DUMMY4(I,IMAX+1)=GLOAD(I)
C      GAUSS JORDAN ELIMINATION
      NUM=IMAX+1
      DO 130 K=1,IMAX
C      CALL WRITE2(DUMMY4)
      LMAX=K
      D2=DUMMY4(K,K)
      AMAX=ABS(D2)
      DO 80 I=K,IMAX
      D1=DUMMY4(I,K)
      IF(AMAX.GE.ABS(D1)) GO TO 80
      LMAX=I
      AMAX=ABS(D1)
80    CONTINUE
      IF(LMAX.EQ.K) GO TO 100
      DO 90 J=1,NUM
      DUMMY6=DUMMY4(LMAX,J)
      DUMMY4(LMAX,J)=DUMMY4(K,J)
      DUMMY4(K,J)=DUMMY6
90    CONTINUE
100   CONTINUE
C      CHECK SINGULAR MATRIX
      D2=DUMMY4(K,K)
      IF(ABS(D2).GT.EPSLON) GO TO 91
      DUMMY4(K,K)=DUMMY4(K,K)+OMEGA
      WRITE(6,2300)K
      STOP
91    CONTINUE
      DDIV=DUMMY4(K,K)
      DO 110 J=1,NUM
      DUMMY4(K,J)=DUMMY4(K,J)/DDIV
110   CONTINUE
      DO 121 I=1,IMAX
      IF(I.EQ.K) GO TO 121
      DMULT=DUMMY4(I,K)
      DO 120 J=K,NUM
      DUMMY4(I,J)=DUMMY4(I,J)-DMULT*DUMMY4(K,J)
120   CONTINUE
121   CONTINUE
130   CONTINUE
      DO 140 I=1,M1
140   GLOAD2(I)=DUMMY4(I,IMAX+1)

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2300  FORMAT(1H0,'KMATRIX BECAME SINGULAR AT ROW',I5)
      RETURN
      END

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      SUBROUTINE STRESS(MAXDEG,MAXEL,MAXID,MAXCOL,ICOL1,GLOAD2,TEMP,IDMT
      XX,ESTOR,GSOL1,ELAST,STOR2)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE FOR    CALCULATING STRESS AND STRAIN IN EACH ELEMENT  C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION ICOL1(MAXDEG),GLOAD2(MAXDEG,1),TEMP(MAXDEG,1),
      XIDMTX(MAXID),ESTOR(MAXEL,6,60),GSOL1(MAXDEG,1),ID(60),DISPL(60,1)
      X,E(6,60),S1(6,1),S2(6,1),ELAST(6,6),A(3,3),V(3,3),D(3),B(3),Z(3)
      DIMENSION STOR2(MAXEL,6)
      EPSL2=0.1D-08
      M1=MAXDEG-MAXCOL
      DO 10 I=1,MAXCOL
10      GSOL1(ICOL1(I),1)=TEMP(I,1)
      DO 11 I=1,M1
11      GSOL1(MAXCOL+I,1)=GLOAD2(I,1)
      DO 20 NEL=1,MAXEL
      NEL2=20*(NEL-1)
      DO 30 I=1,20
      ID(3*I)=3*IDMTX(NEL2+I)
      ID(3*I-1)=ID(3*I)-1
30      ID(3*I-2)=ID(3*I)-2
      DO 40 I=1,60
40      DISPL(I,1)=GSOL1(ID(I),1)
      DO 50 I=1,6
      DO 50 J=1,60
50      E(I,J)=ESTOR(NEL,I,J)
      CALL MULT(6,60,1,E,DISPL,S1)
      CALL MULT(6,6,1,ELAST,S1,S2)
C      GOTO 70
71      CONTINUE
      WRITE(6,1000) NEL
      WRITE(6,1001) S1(1,1),S2(1,1)
      WRITE(6,1002) S1(2,1),S2(2,1)
      WRITE(6,1003) S1(3,1),S2(3,1)
      WRITE(6,1004) S1(4,1),S2(4,1)
      WRITE(6,1005) S1(5,1),S2(5,1)
      WRITE(6,1006) S1(6,1),S2(6,1)
70      CONTINUE
      DO 52 I=1,6
52      STOR2(NEL,I)=S2(I,1)
C      GOTO 20
51      CONTINUE
      A(1,1)=S1(1,1)

```

```

A(2,2)=S1(2,1)
A(3,3)=S1(3,1)
A(1,2)=S1(4,1)
A(1,3)=S1(5,1)
A(2,3)=S1(6,1)
A(2,1)=A(1,2)
A(3,1)=A(1,3)
A(3,2)=A(2,3)
C CALL JACOBI(3,A,V,D,B,Z,IROT,EPSL2)
CALL JACOB2(3,A,V,D,EPSL2)
WRITE(6,1007) D(1),D(2),D(3)
WRITE(6,1008) V(1,1),V(1,2),V(1,3)
WRITE(6,1009) V(2,1),V(2,2),V(2,3)
WRITE(6,1010) V(3,1),V(3,2),V(3,3)
A(1,1)=S2(1,1)
A(2,2)=S2(2,1)
A(3,3)=S2(3,1)
A(1,2)=S2(4,1)
A(1,3)=S2(5,1)
A(2,3)=S2(6,1)
A(2,1)=A(1,2)
A(3,1)=A(1,3)
A(3,2)=A(2,3)
C CALL JACOBI(3,A,V,D,B,Z,IROT,EPSL2)
CALL JACOB2(3,A,V,D,EPSL2)
WRITE(6,1007) D(1),D(2),D(3)
WRITE(6,1008) V(1,1),V(1,2),V(1,3)
WRITE(6,1009) V(2,1),V(2,2),V(2,3)
WRITE(6,1010) V(3,1),V(3,2),V(3,3)
20 CONTINUE
1000 FORMAT(1H0,'ELEMENT',I10)
1001 FORMAT(1H,'EXX',G15.7,10X,'SXX',G15.7)
1002 FORMAT(1H,'EYY',G15.7,10X,'SYY',G15.7)
1003 FORMAT(1H,'EZZ',G15.7,10X,'SZZ',G15.7)
1004 FORMAT(1H,'EXY',G15.7,10X,'SXY',G15.7)
1005 FORMAT(1H,'EXZ',G15.7,10X,'SXZ',G15.7)
1006 FORMAT(1H,'EYZ',G15.7,10X,'SYZ',G15.7)
1007 FORMAT(1H0,'EIGENVALUE',3G15.7)
1008 FORMAT(1H,'EIGENVECTOR-X',3G15.7)
1009 FORMAT(1H,'EIGENVECTOR-Y',3G15.7)
1010 FORMAT(1H,'EIGENVECTOR-Z',3G15.7)
RETURN
END

```

[illegible]

```

DIMENSION A(N,N),V(N,N),D(N)
ITM=200
IT=0
DO 10 I=1,N
DO 10 J=1,N
V(I,J)=0.
10 IF(I.EQ.J) V(I,J)=1.
13 T=0.
M=N-1
DO 20 I=1,M
J1=I+1
DO 20 J=J1,N
IF(DABS(A(I,J)).LE.T) GO TO 20
T=DABS(A(I,J))
IR=I
IC=J
20 CONTINUE
C IF(IT.EQ.0) T1=T*ERR
C IF(T.LE.T1) GOTO 999
IF(T.LE.ERR) GO TO 999
PS=A(IR,IR)-A(IC,IC)
TA=(-PS+DSQRT(PS*PS+4.*T*T))/(2*A(IR,IC))
C=1./DSQRT(1.+TA*TA)
S=C*TA
DO 50 I=1,N
P=V(I,IR)
V(I,IR)=C*P+S*V(I,IC)
50 V(I,IC)=C*V(I,IC)-S*P
I=1
100 IF(I.EQ.IR) GOTO 200
P=A(I,IR)
A(I,IR)=C*P+S*A(I,IC)
A(I,IC)=C*A(I,IC)-S*P
I=I+1
GO TO 100
200 I=IR+1
300 IF(I.EQ.IC) GOTO 400
P=A(IR,I)
A(IR,I)=C*P+S*A(I,IC)
A(I,IC)=C*A(I,IC)-S*P
I=I+1
GO TO 300
400 I=IC+1
500 IF(I.GT.N) GOTO 600
P=A(IR,I)
A(IR,I)=C*P+S*A(IC,I)
A(IC,I)=C*A(IC,I)-S*P
I=I+1
GO TO 500
600 P=A(IR,IR)
A(IR,IR)=C*C*P+2.*C*S*A(IR,IC)+S*S*A(IC,IC)
A(IC,IC)=C*C*A(IC,IC)+S*S*P-2.*C*S*A(IR,IC)
A(IR,IC)=0.
IT=IT+1

```

```
IF(IT.LT.ITM) GOTO 13  
999 DO 900 I=1,N  
900 D(I)=A(I,I)  
RETURN  
END
```